Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II

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Major 2 Exam Term 132 FORM A
Wednesday Mar 19 2014
6.15pm-7.30pm

Name________________________________ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	2+2+2+3+3+3=15		
2	5		
3	4		
4	5		
5	1+2+3=6		
6	1+4=5		
Total	40		

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1. (2+2+2+3+3+3=15 points) PhD students can leave a certain three-year Engineering graduate school only for reasons of failure (Decrement 1) or voluntary withdrawal (Decrement 2), where each decrement is uniformly distributed over $(x,\, x+1)$ in its associated **single-decrement** table. The following values are given:

x	$q_x'^{(1)}$	$q_x'^{(2)}$	$q_x^{(1)}$	$p_x^{(2)}$
0	0.10	0.30	-	-
1	0.20	0.25	-	-
2	0.20	0.15	-	-

Calculate the six missing probability values in the above table.

2. (5 points) The career of a 50 year old Professor of Actuarial Science is subject to two decrements. Decrement 1 is mortality, which is governed by a uniform survival distribution with $\omega = 100$, and Decrement 2 is leaving academic employment, which is governed by the Hazard Rate Function (HRF) $\mu_y^{(2)} = 0.06$, for all $y \geq 50$. Find the probability that this Professor remains in academic employment for **at least** five years but **less than** ten years.

3. (4 points) A whole life contract of face amount 100 000 is issued to (30). The 20^{th} year cash value is 90% of the Net Level Premium (NLP) benefit reserve. The insured has previously borrowed 5000 against the policy at 0% interest. Using the supplied life table with 6% interest, find the **cash value** payable for surrender of the contract at the end of its 20^{th} year.

4. (5 points) The following double-decrement tables gives probability values for a car driver at the beginning of each year in a three-year Motor Insurance policy. Some of the entries in the table have been obliterated by ink stains.

Policy Year	**Probability of Fatal Accident Failure**	**Probability of Voluntary Withdrawal**	**Probability of Completing the Year**
1	0.40	0.20	-
2	-	0.30	-
3	-	-	0.60

It is known that **ten times** as many car drivers **complete** Year 2 as fail during Year 3, and the number of car drivers who fail during Year 2 is 40% of the number who complete Year 2. It is assumed that this probability table is applicable regardless of the driver's age.

Find the probability that a new car driver entering the policy will **voluntarily withdraw** before the end of the policy.

5. (1+2+3=6 points) Consider a five year endowment insurance, with gross annual premiums and annual expenses paid at the beginning of each year and benefits paid at the end of the year. The contingent benefit is 1000 for failure (Decrement 1) within the five year period, or at time $t = 5$ if failure has not previously occurred. A withdrawal benefit (Decrement 2) will be paid in the event of withdrawal from the plan at the end of any of the first four years. All parameters values for the insurance are shown in the following table:

Year k	1	2	3	4	5
Percent of Premium Expense	0.05	0.05	0.05	0.05	0.05
Constant Contract Expense	30	30	30	30	30
Failure Benefit Amount	1000	1000	1000	1000	1000
Withdrawal Benefit Amount	50	100	300	600	0
Endowment Benefit Amount	0	0	0	0	1000
$q_{x+k-1}^{(1)}$	0.02	0.03	0.04	0.05	0.06
$q_{x+k-1}^{(2)}$	0.30	0.20	0.20	0.10	0.00
$q_{x+k-1}^{(\tau)}$	0.32	0.23	0.24	0.15	0.06
$p_{x+k-1}^{(\tau)}$	0.68	0.77	0.76	0.85	0.94

All cash flows are discounted at annual interest rate 4%.
If the contract premium is $G^* = 250.00$ and the initial asset share is ${}_0AS = 70$, find
(1) ${}_5V^G$ (Use an expense augmented premium $G = 184.90$)
(2) ${}_2AS$
(3) Projected surplus U_3 (Use the same expense augmented premium as in part 1)

6. (1+4=5 points) For a fully discrete whole life insurance of 1000 on (x), you are given:

(i) $G = 30$ is the gross premium
(ii) $e_k = 5$, is the per policy expense at the start of year k and $k = 1, 2, 3, ...$
(iii) $c_k = 0.02$, is the fraction of premium expense at the start of year k and $k = 1, 2, 3, ...$
(iv) $i = 0.05$
(v) ${}_4CV = 75$ is the cash value payable upon withdrawal at the end of year 4.
(vi) $q_{x+3}^{(d)} = 0.013$

(vii) $q_{x+3}^{(w)} = 0.05$ (withdrawals occur at the end of the year), and

(viii) ${}_3AS = 25.22$ is the asset share at the end of year 3.

If the probability of withdrawal and all expenses for year 4 are each 120% of the values shown above, by how much does the **asset share** at the end of year 4 decrease?.

a) 1.59
b) 1.64
c) 1.67
d) 1.93
e) 2.03

Work Shown (4 points):

Hence the answer is (   )

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if i = 0.06

Illustrative Life Table

(1989-91 U.S. Life Table, Female Lives)

x	l_x	d_x	p_x	q_x	A_x	${}^2A_x$
0	100,000	828	0.99172	0.00828	0.02525	0.01015
1	99,172	67	0.99932	0.00068	0.01864	0.00315
2	99,105	42	0.99958	0.00042	0.01909	0.00286
3	99,063	32	0.99968	0.00032	0.01982	0.00279
4	99,031	25	0.99975	0.00025	0.02070	0.00282
5	99,006	23	0.99977	0.00023	0.02169	0.00291
6	98,983	21	0.99979	0.00021	0.02277	0.00304
7	98,962	19	0.99981	0.00019	0.02392	0.00321
8	98,943	16	0.99984	0.00016	0.02517	0.00341
9	98,927	16	0.99984	0.00016	0.02653	0.00367
10	98,911	15	0.99985	0.00015	0.02796	0.00396
11	98,896	15	0.99985	0.00015	0.02949	0.00430
12	98,881	18	0.99982	0.00018	0.03111	0.00468
13	98,863	21	0.99979	0.00021	0.03280	0.00508
14	98,842	28	0.99972	0.00028	0.03457	0.00550
15	98,814	34	0.99966	0.00034	0.03637	0.00590
16	98,780	40	0.99960	0.00040	0.03822	0.00628
17	98,740	45	0.99954	0.00046	0.04012	0.00666
18	98,695	48	0.99951	0.00049	0.04209	0.00703
19	98,647	50	0.99949	0.00051	0.04416	0.00741
20	98,597	51	0.99948	0.00052	0.04632	0.00783
21	98,546	54	0.99945	0.00055	0.04861	0.00828
22	98,492	54	0.99945	0.00055	0.05100	0.00876
23	98,438	56	0.99943	0.00057	0.05355	0.00930
24	98,382	57	0.99942	0.00058	0.05622	0.00989
25	98,325	58	0.99941	0.00059	0.05905	0.01053
26	98,267	59	0.99940	0.00060	0.06204	0.01125
27	98,208	61	0.99938	0.00062	0.06520	0.01205
28	98,147	65	0.99934	0.00066	0.06853	0.01293
29	98,082	69	0.99930	0.00070	0.07203	0.01387
30	98,013	74	0.99924	0.00076	0.07570	0.01489
31	97,939	79	0.99919	0.00081	0.07955	0.01599
32	97,860	83	0.99915	0.00085	0.08359	0.01717
33	97,777	88	0.99910	0.00090	0.08783	0.01846
34	97,689	93	0.99905	0.00095	0.09228	0.01986
35	97,596	99	0.99899	0.00101	0.09696	0.02139
36	97,497	104	0.99893	0.00107	0.10186	0.02304
37	97,393	112	0.99885	0.00115	0.10702	0.02485
38	97,281	120	0.99877	0.00123	0.11242	0.02680
39	97,161	128	0.99868	0.00132	0.11808	0.02891
40	97,033	138	0.99858	0.00142	0.12401	0.03121
41	96,895	148	0.99847	0.00153	0.13021	0.03370
42	96,747	161	0.99834	0.00166	0.13671	0.03639
43	96,586	174	0.99820	0.00180	0.14349	0.03929
44	96,412	190	0.99803	0.00197	0.15056	0.04242
45	96,222	210	0.99782	0.00218	0.15794	0.04578
46	96,012	233	0.99757	0.00243	0.16559	0.04936
47	95,779	257	0.99732	0.00268	0.17352	0.05317
48	95,522	282	0.99705	0.00295	0.18174	0.05721
49	95,240	308	0.99677	0.00323	0.19025	0.06151
50	94,932	338	0.99644	0.00356	0.19908	0.06609
51	94,594	372	0.99607	0.00393	0.20820	0.07095
52	94,222	409	0.99566	0.00434	0.21762	0.07609
53	93,813	446	0.99525	0.00475	0.22732	0.08151
54	93,367	486	0.99479	0.00521	0.23734	0.08724
55	92,881	526	0.99434	0.00566	0.24766	0.09331
56	92,355	571	0.99382	0.00618	0.25832	0.09974
57	91,784	621	0.99323	0.00677	0.26930	0.10655
58	91,163	680	0.99254	0.00746	0.28059	0.11372
59	90,483	741	0.99181	0.00819	0.29215	0.12122
60	89,742	803	0.99105	0.00895	0.30398	0.12907
61	88,939	864	0.99029	0.00971	0.31610	0.13730
62	88,075	929	0.98945	0.01055	0.32854	0.14598
63	87,146	999	0.98854	0.01146	0.34130	0.15511
64	86,147	1,072	0.98756	0.01244	0.35438	0.16471
65	85,075	1,146	0.98653	0.01347	0.36778	0.17480
66	83,929	1,222	0.98544	0.01456	0.38151	0.18543
67	82,707	1,302	0.98426	0.01574	0.39560	0.19665
68	81,405	1,391	0.98291	0.01709	0.41005	0.20850
69	80,014	1,492	0.98135	0.01865	0.42483	0.22095
70	78,522	1,603	0.97959	0.02041	0.43987	0.23398
71	76,919	1,722	0.97761	0.02239	0.45514	0.24754
72	75,197	1,847	0.97544	0.02456	0.47060	0.26160
73	73,350	1,972	0.97312	0.02688	0.48621	0.27616
74	71,378	2,091	0.97071	0.02929	0.50200	0.29124
75	69,287	2,205	0.96818	0.03182	0.51800	0.30693
76	67,082	2,318	0.96545	0.03455	0.53425	0.32333
77	64,764	2,443	0.96228	0.03772	0.55079	0.34051
78	62,321	2,588	0.95847	0.04153	0.56752	0.35839
79	59,733	2,747	0.95401	0.04599	0.58431	0.37681
80	56,986	2,909	0.94895	0.05105	0.60102	0.39559
81	54,077	3,061	0.94340	0.05660	0.61756	0.41460
82	51,016	3,196	0.93735	0.06265	0.63389	0.43379
83	47,820	3,308	0.93082	0.06918	0.64999	0.45315
84	44,512	3,397	0.92368	0.07632	0.66588	0.47268
85	41,115	3,472	0.91555	0.08445	0.68153	0.49236
86	37,643	3,530	0.90622	0.09378	0.69682	0.51201
87	34,113	3,540	0.89623	0.10377	0.71158	0.53135
88	30,573	3,498	0.88559	0.11441	0.72582	0.55036
89	27,075	3,409	0.87409	0.12591	0.73957	0.56909
90	23,666	3,294	0.86081	0.13919	0.75282	0.58749
91	20,372	3,141	0.84582	0.15418	0.76533	0.60514
92	17,231	2,921	0.83048	0.16952	0.77684	0.62159
93	14,310	2,638	0.81565	0.18435	0.78742	0.63686
94	11,672	2,326	0.80072	0.19928	0.79729	0.65129
95	9,346	2,007	0.78526	0.21474	0.80659	0.66504
96	7,339	1,698	0.76863	0.23137	0.81533	0.67812
97	5,641	1,398	0.75217	0.24783	0.82338	0.69028
98	4,243	1,119	0.73627	0.26373	0.83087	0.70166
99	3,124	873	0.72055	0.27945	0.83799	0.71258
100	2,251	667	0.70369	0.29631	0.84494	0.72335
101	1,584	498	0.68561	0.31439	0.85170	0.73391
102	1,086	361	0.66759	0.33241	0.85823	0.74420
103	725	256	0.64690	0.35310	0.86476	0.75461
104	469	176	0.62473	0.37527	0.87115	0.76485
105	293	116	0.60410	0.39590	0.87743	0.77493
106	177	74	0.58192	0.41808	0.88424	0.78597
107	103	46	0.55340	0.44660	0.89225	0.79915
108	57	27	0.52632	0.47368	0.90203	0.81554
109	30	15	0.50000	0.50000	0.91670	0.84105
110	15	15	0.00000	1.00000	0.94340	0.89000