

Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies I
Dr. Mohammad H. Omar
Major 1 Exam Term 132 FORM A
Thursday Feb 20 2014
6.00pm-7.15pm

Name _____ ID#: _____ Serial #: _____

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	6		
2	6		
3	6		
4	$3+3+3+3+3=15$		
5	$1+2+3+3+1=10$		
6	$1+5=6$		
Total	40		

Extra blank page

1. (6 points) **Solve** Kolmogorov's differential equation for ${}_n p_{11}^{(y)}$ for a 2-state homogeneous Markov survival model, with constant force of transition function $\lambda_{12}(s) = \lambda_1(s) = \lambda$ and $\lambda_{21}(s) = 0$. Provide the **meaning** of ${}_n p_{11}^{(y)}$ and then translate this into **standard actuarial notation**.

Solution:

2. (6 points) Two microwave brands, denoted Type I and Type II, follow survival models defined by $\mu_x^I = \ln 1.25$, for $x > 0$, and $\mu_x^{II} = \frac{1}{9-x}$, for $0 < x < 9$, respectively. Given that both brands are currently two years old, and that they have independent lifetimes, find the probability that the **first failure** will occur between ages 3 and 6.

Solution:

3. (6 points) A continuous two-life annuity has actuarial present value 1180. The annuity pays at annual rate 100 while both (x) and (y) survive, 70 while (x) survives after the failure of (y) , and 50 while (y) survives after the failure of (x) . Find the value of $\bar{a}_{\overline{xy}}$, given that $\bar{a}_x = 12$ and $\bar{a}_y = 10$.

Solution:

4. (3+3+3+3+3=15 points) Using the **double-decrement** table below,

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(\tau)}$	$p_x^{(\tau)}$	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
45	0.011	0.1	0.111	0.889	1000	11	100
46	0.012	0.1	0.112	0.888	889	10.67	88.90
47	0.013	0.1	0.113	0.887	789.43	10.26	78.94
48	0.014	0.1	0.114	0.886	700.23	9.80	70.02
49	0.015	0.1	0.115	0.885	620.40	9.31	62.04
50	0.016	0.1	0.116	0.884	549.05	8.78	54.91

calculate each of the following

- (a) ${}_3p_{47}^{(\tau)}$
- (b) ${}_2d_{48}^{(2)}$
- (c) ${}_2q_{47}^{(1)}$
- (d) ${}_{2|}q_{46}^{(1)}$
- (e) The probability that a person in the group at age 47 will **leave the group** between ages 49 and 50.

Solution:

5. (1+2+3+3+1=10 points) .Consider the following probability matrix $\mathbf{P} = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Answer the following questions:

(a) What type of Markov Chain model is represented by this transition matrix?

(b) Draw a diagram that shows the way in which the states communicate.

(c) Assume that the model is *homogeneous*. Provide the 2-step transition matrix P^2

(d) If the model is *not homogeneous* where the next transition matrix is given by $\begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

provide the correct value for ${}_2P_{13}^{(0)}$.

(e) If the 4 states are 1 = Normal life, 2 = Sickness, 3 = Disability, and 4 = Death, provide the meaning of ${}_2P_{13}^{(0)}$

6. (1+5=6 points) You are pricing a special 3-year temporary life annuity-due on two lives each age x , with independent future lifetimes, each following the same mortality table. The annuity pays 10,000 if both persons are alive and 2000 if exactly one person is alive.

You are given:

(i) $q_{xx} = 0.04$

(ii) $q_{x+1:x+1} = 0.01$

(iii) $i = 0.05$.

Calculate the expected present value of this annuity and choose the closest answer below.

- a) 27,800
- b) 27,900
- c) 28,000
- d) 28,100
- e) 28,200

Work Shown (5 points):

Solution:

Hence the answer is ()

END OF TEST PAPER