KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 131

STAT 319 Statistics for Engineers and Scientists

Third Major Exam — Monday December 9, 2013

Please check/circle your instructor's name

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Name: Key ID #: __________ Section# ______

☺ Important Notes:

- Show all your work including formulas, intermediate steps and final answer.
- In hypothesis testing problems, write the null and the alternative hypotheses, test statistic, decision rule, critical values, and your conclusion, unless otherwise is specified.

Question No	Full Marks	Marks Obtained
1	4	
2	5	
3	5	
4	6	
5	10	
Total	30	

1) An agricultural economist is interested in determining the average diameter of peaches produced by a particular tree. A random sample of $n = 25$ peaches is taken and the sample mean is calculated. Suppose that the mean diameter of peaches on this tree is known from previous years' production to be 60 mm with a standard deviation of 10 mm.

a) What is the probability that the sample mean exceeds 65 millimeters? *(3 pts.)*

$$P(\bar{x} > 65) = P\left(Z > \frac{(65-60)}{10/\sqrt{25}}\right) = P(Z > 2.5) = 1 - P(Z < 2.5)$$

1pt. 1pt.

$$= 1 - 0.9938 = 0.0062$$

1pt.

b) What assumptions did you make, if any? *(1 pt.)*

Assume that the diameter of a peach has a normal distribution.

2) It is claimed that an automobile in KSA is driven on the average more than 20,000 kilometers per year. To test this claim, a random sample of 100 automobile owners are asked to keep a record of the kilometers they travel. At the 5% significance level would you agree with this claim if the random sample showed an average of 21,000 kilometers and a standard deviation of 3900 kilometers? *(5 pts.)*

1. The hypothesis:

$$H_0: \mu = 20000 \quad vs \quad H_1: \mu > 20000$$

1pt.

2. Since the sample size in large the test statistic

$$Z = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s}$$

$\frac{1}{2}$ pt.

3. The decision rule and the critical value

Reject H_0 if $Z_0 > z_{0.05} = 1.645$ 1 pt.

4. Where

$Z_0 = 2.56$ is the observed test statistic. $\frac{1}{2}$ pt

5. The decision

Since $Z_0 = 2.56 > z_{0.05} = 1.645$, reject H_0 1pt.

6. The conclusion

We conclude that the an automobile in KSA is driven on the average more than 20,000 kilometers per year

1pt.

3) A grinding machine will be qualified for a particular task if it can be shown to produce less than 8% defective parts. In a random sample of 300 parts, 16 were defective. On the basis of these data, can the machine be qualified? Use the p-value approach and discuss all possible decisions. *(5 pts.)*

$$\hat{p} = \frac{16}{300} \quad \frac{1}{2} \text{pt.}$$

The hypothesis:

$$H_0: p = 0.08 \quad vs \quad H_1: p < 0.08 \quad 1 \text{ pt.}$$

1 pt ← The assumptions: $np_0 = 24$ *and* $n(1 - p_0) = 276$

The observed test statistic is

$$Z_0 = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \cancel{-2.05} \; -1.7049 \quad \frac{1}{2} \text{pt.}$$

$$p - value = P(Z < \cancel{-2.05} \; -1.70) = \cancel{0.0202} \; 0.0466 \quad 1 \text{pt.}$$

1 pt For any significance level greater than ~~0.0202~~ 0.0466, we reject the null hypothesis and conclude that the proportion of defective parts is less than 8%, and thus the grinding machine can be qualified.

4) The production manager at a battery factory wants to determine whether there is any difference in the mean life expectancy of batteries manufactured on two different types of machines. A random sample of 21 batteries from machine 1 indicates a mean of 250 hours and a standard deviation of 75 hours, and a similar sample of 21 from machine 2 indicates a mean of 242 hours and a standard deviation of 90 hours. Using the 0.05 level of significance, and assuming that the population variances are equal, is there any evidence of a difference in the mean life of batteries produced by the two types of machines? *(6 pts.)*

1. The hypothesis:

$$H_0: \mu_1 - \mu_2 = 0 \quad vs \quad H_1: \mu_1 - \mu_2 \neq 0 \quad 1 \text{pt.}$$

2. The test statistic

$$\frac{1}{2} \text{pt.} \leftarrow \quad T = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad where \ s_p = 82.8402 \quad 1 \text{pt.}$$

3. The decision rule and the critical value

Reject H_0 if $|T_0| > t_{0.025,40} = 2.0211$ 1 pt.

C.I. = 8 ± 51.67 = [−43.67, 59.67]

4. Where

$$T_0 = \frac{(250-242)}{82.8402\sqrt{\frac{1}{21}+\frac{1}{21}}} = 0.312 \text{ is the observed test statistic} \quad \frac{1}{2} \text{pt.}$$

5. The decision: Since $|T_0| = 0.312 < t_{0.025,40} = 2.0211$, don't reject H_0 1 pt.

6. The conclusion

There is no evidence of a difference in the mean life of batteries produced by the two types of machines

1 pt.

5) In 9 soil specimens tested for trace elements, the average amount of copper was found to be 22 milligrams, with a standard deviation of 4 milligrams.

a) Find a 90% confidence interval for the true mean copper content in the soils from which these specimens were taken. *(3 pts.)*

The 90% confidence interval is given by

1 pt.

1 pt. $\longrightarrow$ $$\bar{x} \pm t_{0.05,8}\frac{s}{\sqrt{n}} \Rightarrow 22 \pm 1.8595\frac{4}{\sqrt{9}}$$

Thus $19.52 \leq \mu \leq 24.48$

1 pt.

b) What assumptions did you make, if any? *(1 pt.)*

Since the sample size is small, we have to assume that the population of the amount of copper present in the soil is normally distributed.

c) Use the confidence interval to test the hypothesis that the mean is 26 mg at the 10% significance level. *(3 pts.)*

$$H_0: \mu = 26 \quad vs \quad H_1: \mu \neq 26$$ 1 pt.

1 pt.

Since $26 \notin (19.52 < \mu < 24.48)$, we conclude that the mean amount of copper is not equal to 26 mg.

1 pt.

d) Find the p-value for testing the hypothesis in c). *(3 pts.)*

The observed test statistic

$$T_0 = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}} = -3$$ 1 pt.

$$p - value = 2P(T_8 > |-3|) = 2P(T_8 > 3)$$ 1 pt.

$$2(0.005) < p - value < 2(0.01)$$

$$0.01 < p - value < 0.02$$ 1 pt.