KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 131

STAT 319 Statistics for Engineers and Scientists

Second Major Exam Monday October 28, 2013

Please check/circle your instructor's name

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Name Key ID # ______________ Section# ________

☺Important Note:

Show all your work including formulas, intermediate steps and final answer.

Question	Full Marks	Marks Obtained
1	21	
2	3	
3	5	
4	6	
5	5	
Total	40	

$IQR = 4\cancel{5}75 - 41.25 = 4.5$

$LOF = Q_1 - 3IQR = 41.25 - 13.5 = 27.75$

$LIF = Q_1 - 1.5IQR = 41.25 - 6.75 = 34.5$

$UIF = Q_3 + 1.5IQR = 45.75 + 6.75 = 52.5$

$UOF = Q_3 + 3IQR = 45.75 + 13.5 = 59.25$

$\Rightarrow$ 55 is a mild (possible) outlier.

1) The following data are the temperatures of effluent at discharge from a sewage treatment facility on consecutive days:

36	39	41	41	42	42	42	43
44	44	44	45	46	46	52	55

a) Calculate the mean, median, mode and variance. (You may want to use $\sum x = 702$, and $\sum x^2 = 31114$) *(4pts.)*

Mean $\bar{x} = \frac{702}{16} = 43.875$ 1

Median $= \frac{43+44}{2} = 43.5$ 1

Mode = 42 and 44 1

Variance $s^2 = \frac{1}{15}\left[31114 - \frac{(702)^2}{16}\right] = 20.916$ 1

b) Use the information above to comment on the shape of the data. *(1pt.)*

Since the mean is greater than the median then the data is skewed to the right (or positively skewed).

c) Is the empirical rule satisfied? Explain. *(3pts.)*

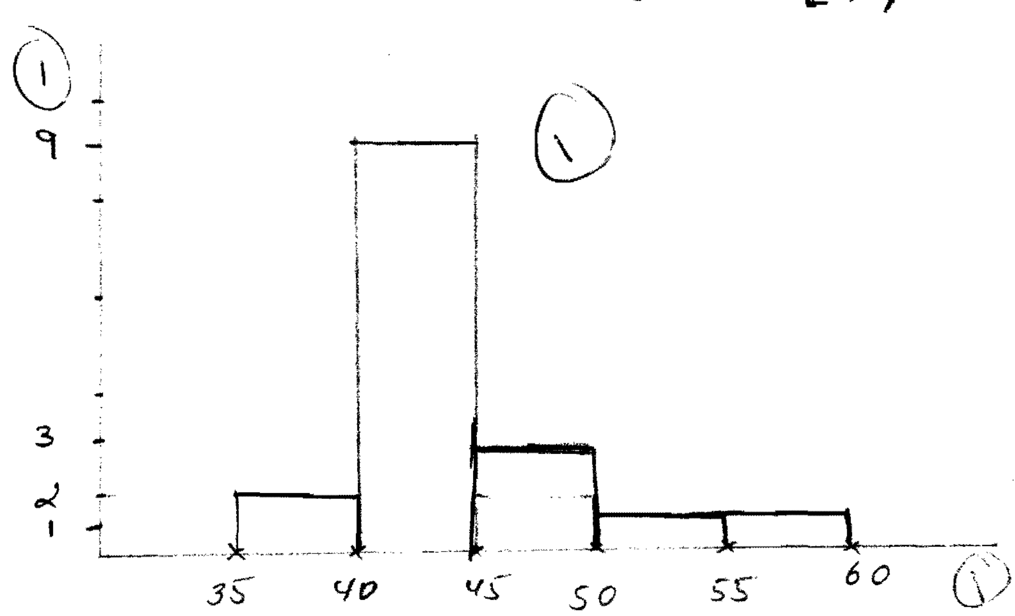
$S = 4.573$;

$\bar{x} \pm s \Rightarrow [39.677, 48.823]$ 1

%age of data within one standard deviation is $\frac{12}{16} \times 100 = 75\%$ 1

Thus the empirical rule is not satisfied 1

d) Construct a frequency histogram including the interval [40,45). *(3pts.)*



e) From part d), approximate the mean of the data. *(2pts.)*

Approximate Mean $= \frac{1}{16}\left[(37.5\times 2)+(42.5\times 9)+(47.5\times 3)+(52.5\times 1)+(57.5\times 1)\right]$ (1)

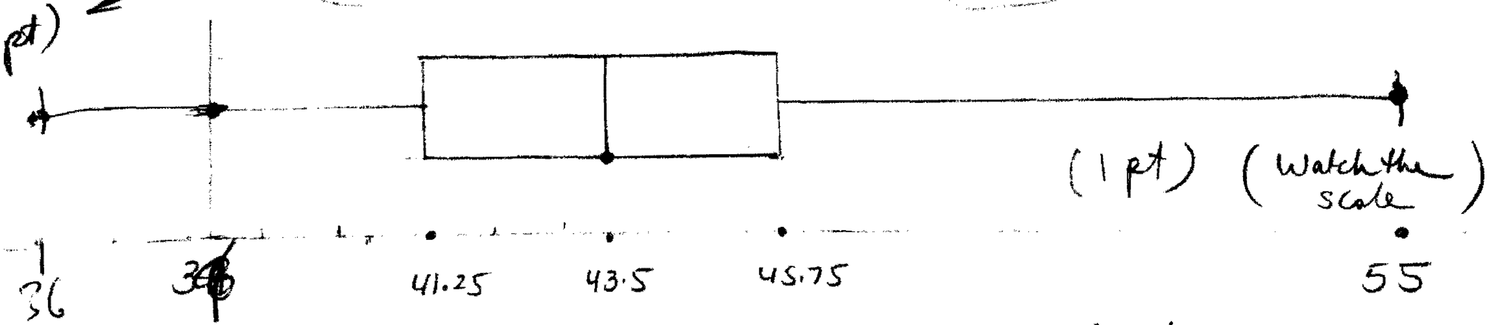
$= \frac{710}{16}$

$= 44.375$ (1)

Class	f	x	xf
[35,40)	2	37.5	75
[40,45)	9	42.5	382.5
[45,50)	3	47.5	142.5
[50,55)	1	52.5	52.5
[55,60)	1	57.5	57.5
Total	16		

f) Construct a box plot of the data, and comment. *(5pts.)*

(1pt) $Q_1 = 41.25$, $Q_2 = 43.5$, $Q_3 = 45.75$ (1pt)



(1 pt) (Watch the scale)

(1pt) - The middle 50% of the data is relatively tight

(1pt) - The data seem to be skewed to the right

g) Find 90th percentile and explain its meaning in terms of the temperature of sewage discharge. *(3pts.)*

Position of 90th percentile $P_{0.9}$ is

(1pt) $(0.90)\times 17 = 15.3$

Thus the 90th percentile is $52 + 0.3(55-52)$

(1pt) $= 52.9 = 0.7(52) + 0.3(55)$

∴ 90% of the effluent discharge has temperature less than 52.9

2) The proportion of impurities Y in a batch of product of a chemical process has the density function

$$f(y) = \begin{cases} 10\,(1-y)^9 & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

A batch is considered not acceptable if the percentage of impurities exceeds 60%. What is the percentage of batches that are not acceptable? *(3pts.)*

$P(\text{Batch is not acceptable}) = P(Y > 0.6)$ (1pt)

$= \int_{0.6}^{1} 10(1-y)^9\,dy$

$= (1-y)^{10} \Big|_{0.6}^{1}$

$= (0.4)^{10}$ (1pt)

$= 1.04 \times 10^{-4}$

$= 1.04857 \times 10^{-4} = 0.000105$

(1pt) Thus % age of batches that are not acceptable is 0.01

Percentage $= 0.000105 \times 100 = 0.0105\%$

3) The reliability of an electrical fuse is the probability that a fuse, chosen at random from production, will function under its designed conditions. A random sample of 1000 fuses was tested and 27 defectives were observed. Calculate the approximate probability of observing 27 or more defectives, assuming that the fuse reliability is 0.98.

(5pts.)

(1pt) $\left\{ \begin{array}{l} \text{Let } X = \#\text{ of defective fuses.} \\ X \sim B(1000, 0.02) \end{array} \right.$; (1pt) $\mu_X = 20$, (1pt) $\sigma_X = 4.4272$

(1pt)

$P(X \geq 27) \cong P\left(Z \geq \dfrac{27 - 20 - 1/2}{4.43}\right)$

$= P(Z \geq \overset{1.47}{1.467}) = P(Z \leq -1.47)$

$= 1 - \cancel{0.9289}\ 0.9292$

$= 0.0708$ (1pt)

4) Light bulbs produced by a certain manufacturer have a useful life that is normally distributed with a mean of 250 hours and a variance of 2500.

a) What is the probability that a randomly selected bulb from this production process will have a useful life between 190 and 270 hours? *(4pts.)*

X: life of light bulb ; $X \sim N(250, 2500)$

$P(190 \le X \le 270) = P\left(\frac{190-250}{50} \le Z \le \frac{270-250}{50}\right)$ (1pt)

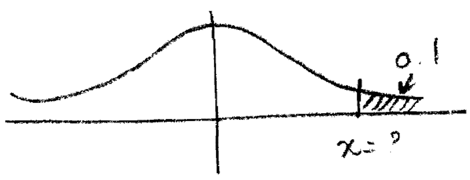
$= P(-1.2 \le Z \le 0.4)$

$= P(Z \le 0.4) - P(Z \le -1.2)$

(1pt) $= 0.6554 - 0.1151$ (1pt)

$= 0.5403$ (1pt)

b) Find the number of hours that only 10% of the bulbs live longer than. *(2pts.)*



To find x such that $P(X \le x) = 0.9$ (1pt)

$P(Z \le 1.28) = 0.9 \Rightarrow 1.28 = \frac{x-250}{50}$

$\Rightarrow x = 314$ hours (1pt)

5) The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes.

a) What is the probability that a person is served in less than 3 minutes? *(2pts.)*

X has pdf $f(x) = \frac{1}{4} e^{-x/4}$; $x > 0$ when X = service time.

$P(X < 3) = \int_0^3 \frac{1}{4} e^{-x/4}\,dx = -e^{-x/4}\Big|_0^3 = 1 - e^{-3/4}$

$= 0.5276$ (1pt)

(1pt)

b) What is the median time of service? *(3pts.)*

Let m = median time

(1pt) $\frac{1}{2} = \int_0^m \frac{1}{4} e^{-\frac{x}{4}}\,dx = 1 - e^{-m/4} \Rightarrow e^{-m/4} = 1/2 \Rightarrow$ (1pt) $m = 4 \ln 2$ (1pt)

$= 2.77$ minutes