KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 131

STAT 319 Statistics for Engineers and Scientists

First Major Exam — Monday September 30, 2013

Please check/circle your instructor's name

□ Abbasi □ Anabosi □ Jabbar □ Al-Sabah □ Saleh □ Alsawi

Name: Key ID #: ____________ Section# ______

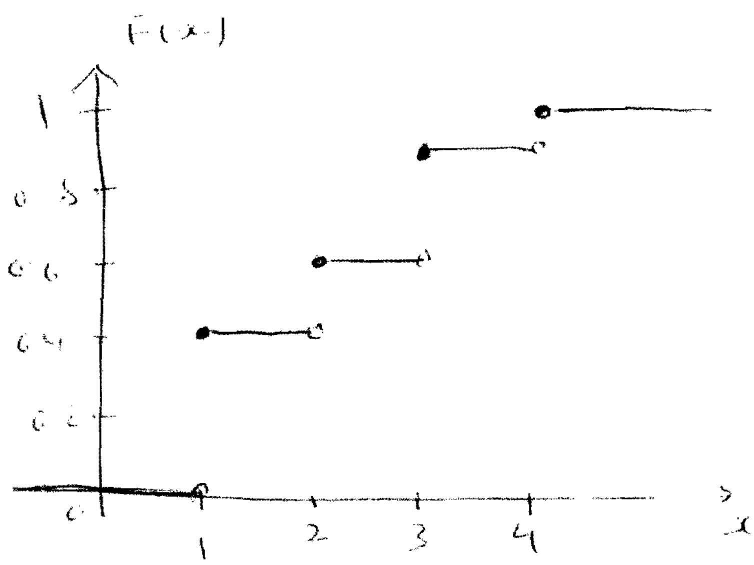
☺Important Note:

Show all your work including formulas, intermediate steps and final answer.

Question No	Full Marks	Marks Obtained
1	5	
2	6	
3	4	
4	4	
5	6	
Total	25	

Some Useful Formulas

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) > 0$
- $P(E_1 \mid B) = \dfrac{P(B \mid E_1) P(E_1)}{P(B \mid E_1) P(E_1) + \cdots + P(B \mid E_k) P(E_k)}$ for $P(B) > 0$
- $f(x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, 2, \ldots, n$
- $f(x) = \dfrac{\dbinom{N-K}{n-x}\dbinom{K}{x}}{\dbinom{N}{n}}$, $x = 1, 2, \cdots, \min(n, K)$
- $f(x) = p\,(1-p)^{x-1}$, $x = 1, 2, \ldots$
- $f(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \cdots$



1) A critical automobile part is inspected by three different inspectors having rejection rates of 0.10, 0.08, and 0.12, respectively. The inspections are independent and sequential such that if a part is rejected by one inspector it is immediately removed.

a) What is the probability that a part never reaches the third inspector? *(3 pts.)*

P(part never reaches 3^{rd} inspector)

= P(part is rejected by 1^{st} inspector or accepted by 1^{st} and rejected by 2^{nd}) (1 pt)

$= 0.10 + (0.90)(0.08)$ (1 pt)

$= 0.1072 = 0.172$ (1 pt)

b) What is the probability that a part is rejected by the third inspector? *(2 pts.)*

P(part is rejected by 3^{rd})

= P(part is accepted by 1^{st} and accepted by 2^{nd} and rejected by 3^{rd}) (1 pt)

$= (0.9)(0.92)(0.12) = 0.09936$ (1 pt)

[1 − 0.172]

2) A chemical supply company ships a certain solvent in 10–gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume X has the following probability mass function:

x	1	2	3	4
$P(X = x)$	0.4	0.2	0.3	0.1

a) Find the cumulative distribution function of X. *(4 pts.)*

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \quad \leftarrow (1\text{ pt}) \\ 0.4 & 1 \le x < 2 \\ 0.6 & 2 \le x < 3 \quad \leftarrow (1\text{ pt}) \\ 0.9 & 3 \le x < 4 \quad \leftarrow (1\text{ pt}) \\ 1.0 & x \ge 4 \quad \leftarrow (1\text{ pt}) \end{cases}$$

graph

b) Find the mean number of gallons ordered. *(2 pts.)*

Mean number of drums $= \sum x P(X = x)$ (1 pt)

$= 1(0.4) + 2(0.2) + 3(0.3) + 4(0.1)$

$= 2.1$ drums

Thus mean # of gallons $= 21$ gallons. ← (1 pt)

3) There is a 10% chance that an electric fuse is defective. A quality controller picks 4 fuses at random from a large batch and tests each one.

a) What is the probability of finding at least one defective fuse? *(2pts.)*

If X = # of defective fuses

$$P(X \geq 1) = 1 - P(X=0) \quad (1\text{ pt})$$

$$\left.\begin{aligned} &= 1-(0.9)^4 \\ &= \boxed{0.3439} \end{aligned}\right\} (1\text{ pt})$$

b) What is the probability that the first defective fuse is the last one tested? *(2pts.)*

P(4th fuse is the 1st defective fuse)

$$=(0.9)^3(0.1) \quad (1\text{ pt})$$

$$= \boxed{0.0729} \quad (1\text{ pt})$$

4) The number of oil tankers arriving at a certain refinery each day has a Poisson distribution with rate equal to 2. Present port facilities can service three tankers a day. If more than three tankers arrive in a day, the tankers in excess of three must be sent to another port. On a given day what is the probability of having to send tankers away? *(4pts.)*

X = # of oil tankers

$$P(X=x) = \frac{e^{-2}\,2^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$P(\text{send tankers away}) = P(X>3) \quad (1\text{ pt})$$

$$= 1 - P(X \leq 3) \quad (1\text{ pt})$$

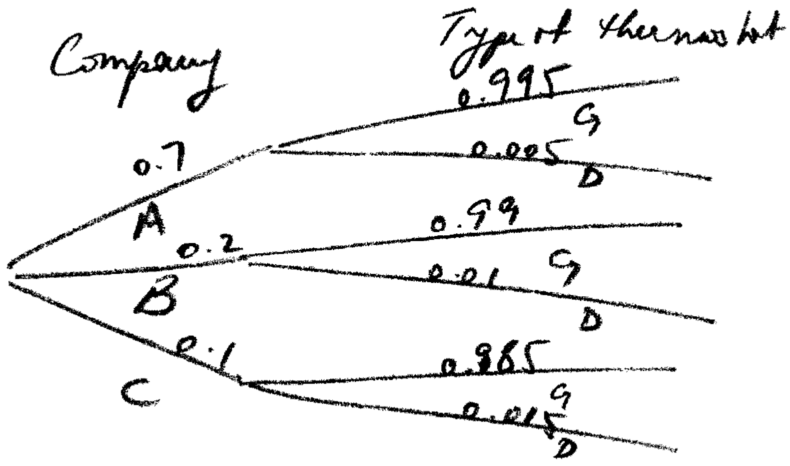
$$\left.= 1 - e^{-2}\left[1+2+2+\frac{8}{6}\right]\right\} (1\text{ pt})$$

$$= 1 - 0.857$$

$$= \boxed{0.142} \leftarrow (1\text{ pt})$$

5) A manufacturer of air-conditioning units purchases 70% of its thermostats from company A, 20% from company B, and the rest from company C. Past experience shows that 0.5% of company A's thermostats, 1% of company B's thermostats and 1.5% of company C's thermostats are likely to be defective. An air-conditioning unit is randomly selected from this manufacturer's production line.

a) Find the probability that the selected thermostat is defective. *(4pts.)*



$(0.7)(0.005)$

$(0.2)(0.01)$

$(0.1)(0.015)$

(2pts)

(1pt)

$P(\text{Thermostat is defective}) = (0.7)(0.005) + (0.2)(0.01) + (0.1)(0.015)$

$= \boxed{0.007}$ (1pt)

b) Find the probability that company A supplied the defective thermostat. *(2pts.)*

$P(A \text{ supplied the thermostat} \mid \text{thermostat is defective})$

$= \dfrac{(0.7)(0.005)}{0.007}$ (1pt)

$= \boxed{0.5}$ (1pt)