Exam I- Math 572- Term 131

Name:

Q: Consider the boundary value problem:

$$\begin{aligned} -u''(x) + u'(x) + u(x) &= 1 \quad in \quad (c, d), \\ u'(c) = \alpha, \quad u(d) = \beta. & \end{aligned}$$

Find a suitable transformation T,

$$T : [c, d] \longrightarrow [0, 1]$$

to transform the above problem to

$$\begin{aligned} -au''(x) + bu'(x) + u(x) &= f(x) \quad in \quad (0, 1), \\ u'(0) = u_0, \quad u(1) = u_1. & \end{aligned}$$

Q: Let $\mathbf{U} = [u_1, u_2]^t$ and c be smooth functions defined in R^2,

$$\begin{aligned} -\nabla c(\mathbf{x}) + c(\mathbf{x})\mathbf{U}(\mathbf{x}) &= [\sin x_2, \cos x_1]^t, \\ \nabla \cdot \mathbf{U}(\mathbf{x}) &= 0. \end{aligned}$$

Show that

$$-\Delta c(\mathbf{x}) + \mathbf{U}(\mathbf{x}) \cdot \nabla c(\mathbf{x}) = 0.$$

Q: Let $\Omega \subset R^n$ be an open, bounded set with smooth boundary Γ. Let also $\mathbf{n}(\mathbf{x})$ be the outward unit normal to Γ and f, g some smooth functions defined on $\overline{\Omega}$. Show that the boundary value problem:

Find $c : \Omega \to R$ satisfying

$$\begin{aligned} -\Delta c &= f \;\; in \;\; \Omega, \\ -\mathbf{n} \cdot \nabla c &= g \;\; on \;\; \Gamma, \end{aligned}$$

has no solution if

$$\int_\Omega f(\mathbf{x}) d\mathbf{x} \neq \int_\Gamma g(\mathbf{s}) d\mathbf{s}.$$

Q: Write a finite difference scheme for the boundary value problem:

$$\begin{aligned} -u''(x) &= 1 \qquad in \;\; (0,4), \\ u(0) &= \alpha, \qquad u'(4) = \beta u(4). \end{aligned}$$

Here, α and β are constants.

Q: Consider the boundary value problem:

$$\begin{aligned} -u''(x) + u'(x) &= f(x) \quad in \quad (0,1), \\ u(0) = \alpha, \quad u(1) &= \beta. \end{aligned}$$

(1) Use the centered finite differences formulas for $u'(x)$ and $u''(x)$ to approximate the above problem.
(2) Derive the discrete maximum principle.

(3) Derive the discrete stability estimate.
(4) Show that the discrete problem has a unique solution.

Q: Prove that the Green's function G for the boundary value problem

$$\begin{aligned} -u''(x) + u'(x) &= f(x) \quad in \quad (0,1), \\ u(0) = 0, \quad u(1) &= 0, \end{aligned}$$

is symmetric: $G(x,y) = G(y,x)$.