Math 513-131 HW 1

Name:..Sec#:................ID#:..............................Ser#:.........

Q.1: Find Fourier series of the function $f(t) = \begin{cases} 0 & -\pi < t < 0 \\ \sin(t) & 0 < t < \pi \end{cases}$.

Use the result to show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \ldots$. Plot the graph of function with partial sums of the series for $n = 1, 2, \ldots, 6$.

Sol: $a_0 = \dfrac{1}{\pi}\int_0^\pi \sin(t)\ dt = \dfrac{2}{\pi}$.

For $n \neq 1$, $a_n = \dfrac{1}{\pi}\int_0^\pi \sin(t)\cos(nt)\ dt = \dfrac{1}{2\pi}\int_0^\pi [\sin(1+n)t + \sin(1-n)t]\ dt$

$$= \frac{-1}{2\pi}\left[\frac{\cos(1+n)t}{1+n} + \frac{\cos(1-n)t}{1-n}\right]_0^\pi = \frac{-1}{2\pi}\left[\frac{\cos(n+1)t}{n+1} - \frac{\cos(n-1)t}{n-1}\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[\frac{(-1)^{n+1}}{n+1} - \frac{(-1)^{n-1}}{n-1} - \frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= \frac{1}{2\pi}\left[\frac{(-1)^n}{n+1} - \frac{(-1)^n}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{-1}{\pi}\left[\frac{(-1)^n}{n^2-1} + \frac{1}{n^2-1}\right] = \frac{-1}{\pi}\left[\frac{(-1)^n + 1}{n^2-1}\right]$$

$$a_1 = \frac{1}{\pi}\int_0^\pi \sin(t)\cos(t)\ dt = \frac{1}{\pi}\left[\frac{\sin^2(t)}{2}\right]_0^\pi = 0.$$

For $n \neq 1$, $b_n = \dfrac{1}{\pi}\int_0^\pi \sin(t)\sin(nt)\ dt = \dfrac{1}{2\pi}\int_0^\pi [\cos(1-n)t - \cos(1+n)t]\ dt$

$$= \frac{1}{2\pi}\left[\frac{\sin(1-n)t}{1-n} - \frac{\sin(1+n)t}{1+n}\right]_0^\pi = \frac{-1}{2\pi}\left[\frac{\cos(n+1)t}{n+1} - \frac{\cos(n-1)t}{n-1}\right]_0^\pi = 0$$

$$b_1 = \frac{1}{\pi}\int_0^\pi \sin^2(t)\ dt = \frac{1}{2\pi}\int_0^\pi [1 - \cos(2t)]\ dt = \frac{1}{2}.$$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\sin(t) - \frac{1}{\pi}\sum_{n=2}^{\infty}\left[\frac{(-1)^n+1}{n^2-1}\right]\cos(nt).$$

Q.2: Find Half Range Fourier sine and cosine series of $f(t) = \pi^2 - t^2$, $0 < t < \pi$. Write these series in Phase angle form. Plot the graph of function with partial sums of the series for $n = 1, 2, \ldots, 6$.

Sol: $a_0 = \dfrac{2}{\pi}\int\limits_0^{\pi} \pi^2 - t^2 \; dt = \dfrac{4\pi^2}{3}$.

$$a_n = \frac{2}{\pi}\int\limits_0^{\pi} (\pi^2 - t^2)\cos(nt)\; dt = \frac{4[0 - n\cos(n\pi)\pi]}{\pi n^3} = \frac{-4(-1)^n}{n^2}$$

Half Range Fourier Cosine Series $f(t) = \dfrac{2\pi^2}{3} - \sum\limits_{n=1}^{\infty}\left[\dfrac{4(-1)^n}{n^2}\right]\cos(nt)$

$$b_n = \frac{2}{\pi}\int\limits_0^{\pi} (\pi^2 - t^2)\sin(nt)\; dt = \frac{2(\pi^2 n^2 + 2 - 2(-1)^n)}{\pi n^3}$$

Half Range Fourier Sine Series $f(t) = \sum\limits_{n=1}^{\infty}\left[\dfrac{2(\pi^2 n^2 + 2 - 2(-1)^n)}{\pi n^3}\right]\sin(nt)$

Note: a_n and b_n are calculated using integration by parts.