King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam 1
The First Semester of 2013-2014 (131)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		12
3		12
4		12
5		12
6		12
7		14
8		14
Total		100

Q:1 (12 points) Find the Fourier series of $f(t) = t \sin\left(\frac{\pi t}{L}\right)$ for $-L < x < L$.

Q:2 (12 points) Find the complex Fourier series of $f(x) = t^2 - \pi t$ for $-\pi \leq x \leq \pi$.

Q:3 (12 points) Show that Fourier transform of $f(t) = \begin{cases} e^{-(1+i)t} & t > 0 \\ -e^{(1-i)t} & t < 0 \end{cases}$,

is $F(w) = \dfrac{-2i(w+1)}{(w+1)^2+1}$.

Q:4 (12 points) $\mathcal{F}\{e^{-2t}H(t)\} = \dfrac{1}{2+iw}$ and Parseval's equality to show that $\int\limits_{-\infty}^{\infty} \dfrac{1}{x^2+4}\,dx = \dfrac{\pi}{2}$.

Q:5 (12 points) Let $f(t) = H(t+2) - H(t-2)$ and $g(t) = e^{-t}H(t)$.

Show that $f(t) * g(t) = \begin{cases} 0 & t \le -2 \\ 1 - e^{-(t+2)} & -2 \le t \le 2 \\ e^{-(t-2)} - e^{-(t+2)} & 2 \le t \end{cases}$.

Q:6 (12 points) Solve the initial value problem by using Laplace transform,

$$y'' + 3y' + 2y = e^t H(t-1),\ y(0) = y'(0) = 0.$$

Q:7 (7+7 points) (a) Find laplace transform of $f(t) = te^{2t}\cos(3t)$.

(b) Find inverse laplace transform of $F(s) = \dfrac{(2s+3)e^{-\pi s}}{s^2+4} + \dfrac{s}{s^2+6s+13}$.

Q:8 (7+7 points) (a) Use convolution to find inverse laplace transform of $F(s) = \dfrac{1}{(s^2+1)^2}$.

(b) Verify the laplace convolution theorem for $t^2 * \sin(2t) = \dfrac{t^2}{2} - \dfrac{1}{2}\sin^2(t)$.