FINAL EXAM

MATH 411

Prob: 1

Let $f(x) = x^{-1}(\log x)^{-2}$, $A = (0, 1/2)$. Show that $f \in L^p(A)$ for $p=1$, but not for $p>1$.

Prob. 2

Let $f_\nu(x) = \frac{\nu}{x^2+\nu^2}$, $\nu = 1, 2, \ldots$ Show that $0 \le f_\nu(x) \le 1$, $\lim_{\nu \to \infty} f_\nu(x) = 0$ for every x and $\int f_\nu \, dx = \pi$. Why does this not contradict the Dominated Convergence Theorem.

Prob 3:

Let $g(s,t) = e^s \cos t \, e_1 + e^s \sin t \, e_2$ with e_1 and e_2 as a basis. Prove that g is not univalent and is univalent on $\Delta = \{(s,t) ; 0 < t < 2\pi\}$. Find its inverse.

Prob 4:

Prove that A is measurable iff $\forall \varepsilon > 0$ there exists a compact set K and an open set G s.t. $K \subset A \subset G$ and $V(G \setminus K) < \varepsilon$.

Prob 5:

Let $\Phi(x,y,z) = x^2 + 4y^2 - 2yz - z^2$, $x_0 = 2e_1 + e_2 - 4e_3$

a) Verify the hypotheses of the implicit fct. theorem.

b) Find the largest neighborhood U of x_0 s.t. $\Phi_3(x,y,z) \neq 0$ $\forall (x,y,z) \in U$.

c) Find the largest neighborhood of x_0 (s.t. $\Phi_3(x,y,z) \neq 0$) containing no critical pt of Φ.