KFUPM **Semester 131**

Dept. Math. &Stat. **A.Y:2013/2014**

Name: ....

ID: ....

Exercise 1.

Express the following in the form $x + iy$:

$$\frac{(\sqrt{3}-i)^2(1+i)^5}{(\sqrt{3}+i)^4}$$

[Use polar forms of complex numbers]

Exercise 2.

For a fixed positive integer n, determine the real part of $(1 + i\sqrt{3})^n$.

Exercise 3.

Find two complex numbers z_1 and z_2 so that

$$\operatorname{Arg}(z_1 z_2) \neq \operatorname{Arg} z_1 + \operatorname{Arg} z_2.$$

(where Arg(z) is the principal argument of z)

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Exercise 4.

Let f be the complex function defined for $z = x + iy$ by

$$f(z) = \begin{cases} \dfrac{xy^2}{x^2 + y^2} & \text{if } z \neq 0, \\ 0 & \text{if } z = 0. \end{cases}$$

a. Show that Cauchy-Riemann equations are satisfied for f only at the origin 0.

b. Is f differentiable at 0 ?

Exercise 5. Let $f(z)$ be a complex function defined on $\mathbb{C}$.

Show that if $f(z)$ and $\overline{f(z)}$ are entire functions, then $f(z)$ is constant.