KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: FINAL EXAM, SEMESTER (131), JANUARY 06, 2014

Time: 07:00 to 10:00 pm

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Name : ....

ID : .... Section : ....

Exercise	**Points**
1	____ / 8
2	____ / 8
3	____ / 8
4	____ / 8
5	____ / 8
6	____ / 22
7	____ / 18
8	____ / 20
9	____ / 22
10	____ / 18
Total	____ / 140

(Part I): Multiple Choice Questions

In this part, write which of the statements a, b, c or d is true in the answer box provided (only one statement is correct).

Exercise 1 (8 pts)**.** If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$, then there exists an invertible matrix P such that $P^{-1}AP$ is equal to

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Ans: ☐

Exercise 2 (8 pts)**.** Let $F(x, y, z) = x^3\mathbf{i} + x^2y\mathbf{j} + x^2z\mathbf{k}$ and S be the surface consisting of the cylinder $(x^2 + y^2 = 4;\ 0 \le z \le 2)$ and the circular diks $(x^2 + y^2 \le 4;\ z = 0)$ and $(x^2 + y^2 \le 4;\ z = 2)$. The flux $\iint_S F.\mathbf{n}\, \mathrm{d}S$ of F across the surface S is equal to

(a) 45π

(b) 20π

(c) 25π

(d) 40π

Ans: ☐

Exercise 3 (8 pts)**.** If $z = \dfrac{1+i\sqrt{3}}{1+i}$, then z^{12} is equal to

(a) -2^5

(b) -2^6

(c) $i2^5$

(d) $i2^6$

Ans: ☐

Exercise 4 (8 pts)**.** Recall that the if $z \in \mathbb{C} \setminus \{0\}$and $\alpha \in \mathbb{C}$, then the principal value of the power z^α is the complex number $e^{\alpha \mathrm{Ln}(z)}$, where $\mathrm{Ln}(z)$ is the principal logarithm of z.

The principal value of the power $(1+i\sqrt{3})^i$ is

(a) $e^{-\frac{\pi}{3}}\left(\cos(\log_e 2) + i\sin(\log_e 2)\right)$

(b) $e^{-\frac{\pi}{6}}\left(\cos(\log_e 2) + i\sin(\log_e 2)\right)$

(c) $e^{-\frac{\pi}{6}}\left(\cos(2\log_e 2) + i\sin(2\log_e 2)\right)$

(d) $e^{-\frac{\pi}{3}}\left(\cos(2\log_e 2) + i\sin(2\log_e 2)\right)$

Ans: ☐

Exercise 5 (8 pts)**.** Let $\mathcal{C}$ be the positively oriented circle given by $|z-i| = 1$. The value of $\oint_{\mathcal{C}} \dfrac{e^{z^2}}{z-i}\, \mathrm{d}z$ is

(a) $\dfrac{4\pi i}{e}$

(b) $\dfrac{4\pi i}{e^2}$

(c) $\dfrac{2\pi i}{e^2}$

(d) $\dfrac{2\pi i}{e}$

Ans: ☐

(Part II): Written Questions

Exercise 6 (22 pts)**.**

(a) Find all complex numbers z such that
$$\sin(4z) = \frac{i}{2}\cos(4z).$$

(b) Write

$$\sinh(1 + i\frac{\pi}{3}) + \sec(\pi + i)$$

in the form $a + ib$.

Exercise 7 (18 pts)**.** Let $f(z) = |z|^2 + \overline{z}$. We denote by $x = \text{Re}(z)$, $y = \text{Im}(z)$, $u(x, y) = \text{Re}(f(z))$ and $v(x, y) = \text{Im}(f(z))$.

(a) Find all (x, y) at which u, v satisfy Cauchy-Riemann Equations.

(b) Is $f(z)$ differentiable at $z_0 = -1$? Why?

Exercise 8 (20 pts)**.** Using Cauchy integral formulas, evaluate the contour integral

$$\oint_{\mathcal{C}} \left[\frac{\cosh(z)}{z^3} - \frac{\sin^2 z}{(2z-\pi)^3} + \frac{e^z}{z-3} \right] \, \mathrm{d}z,$$

where $\mathcal{C}$ is the positively oriented circle $|z| = 2$.

Exercise 9 (22 pts)**.** Let $f(z) = \dfrac{z^3 - 4z^2 + 5z}{z^2 - 4z + 3}$.

(a) Find the Laurent series of the function $f(z)$ about $z_0 = 1$ in the region $0 < |z - 1| < 2$.

(b) Use (a) to evaluate the contour integral

$$I = \oint_{\mathcal{C}} f(z)\,\mathrm{d}z,$$

where $\mathcal{C}$ is the positively oriented circle $|z - \frac{3}{2}| = 1$.

Exercise 10 (18 pts)**.** Consider the function $f(z) = \dfrac{e^{iz} + \cos(z)}{z(z-\pi)^2}$.

(a) Find $\operatorname{Res}(f(z), 0)$ and $\operatorname{Res}(f(z), \pi)$.

(b) Use (a) to evaluate $\oint_{\mathcal{C}} f(z)\,\mathrm{d}z$

where $\mathcal{C}$ is the positively oriented circle $|z-3| = 1$.