KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: EXAM II, SEMESTER (131), NOVEMBER 25, 2013

Time: 08:00 to 10:00 pm

Name : ....

ID : .... Section : ....

Exercise	Points
1	____ /10
2	____ /10
3	____ /20
4	____ /20
5	____ /20
6	____ /20
Total	____ /100

Exercise 1. Let $f(x, y) = x^2 + y$.

(a) Find the directional derivative of $f(x, y)$ in the direction of the vector $\mathbf{v} = (1, 1)$.

(b) Find the equations of the normal line and the tangent plane to the surface $z = f(x, y) = x^2 + y$ at the point $(0, 1, 1)$.

Exercise 2. Let $\mathcal{C}$ be the curve given by $y = x^2 + 1$, from $(0,1)$ to $(2,5)$.
Evaluate the line integral with respect to arc length

$$I = \int_{\mathcal{C}} x^2 \mathrm{ds}$$

Exercise 3. Let $\mathcal{C}$ be the positively oriented boundary of the region in the first quadrant consisting of all points between the two curves $x^2+y^2=1$ and $x^2+y^2=4$.

(a) Graph the path $\mathcal{C}$.

(b) Using Green's theorem, evaluate the work done by the force

$$F(x,y) = (\frac{1}{3}y^3 - \sin(x)e^{x^2})\mathbf{i} + (xy + xy^2 + y^3\cos(y))\mathbf{j}$$

in moving a particle along the path $\mathcal{C}$.

Exercise 4. Let F be the vector field given by

$$F(x,y) = (ye^{xy} + y^2 - 6xy + 6)\mathbf{i} + (xe^{xy} + 2xy - 3x^2 - 2y)\mathbf{j}.$$

(a) Show that F is conservative.

(b) Find a potential of F.

(c) Evaluate the line integral $\int_{\mathcal{C}} F.\mathrm{dr}$, where $\mathcal{C}$ is the positively oriented quarter circle joining $(2,0)$ and $(0,2)$.

Exercise 5. Let $\mathcal{S}$ be the portion of the surface $z = x^2 - y^2$ in the first octant that is within the cylinder $x^2 + y^2 = 9$.

(a) Find the surface area of $\mathcal{S}$.

(b) Evaluate the surface integral

$$\iint\limits_{S} \frac{xy}{x^2+y^2}\ \mathrm{dS}$$

Exercise 6. Let $\mathcal{S}$ be the surface given by

$$z = (1-x)^2, \ \ 0 \leq x \leq 1, \ \ -1 \leq y \leq 1.$$

Suppose that $\mathcal{S}$ is upward oriented and let $\mathcal{C}$ be the boundary of $\mathcal{S}$. The path $\mathcal{C}$ has a positive orientation with respect to the upward orientation of $\mathcal{S}$.

Verify Stokes theorem

$$\oint_{\mathcal{C}} F.\mathrm{dr} = \iint_{S} (\mathrm{Curl}(F).\mathbf{n}) \ \mathrm{dS},$$

where $F(x, y, z) = (xy, yz, xz)$.