KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: EXAM I, SEMESTER (131), OCTOBER 05, 2013

Time: 08:00 to 09:30 am

Name : ....

ID : .... Section : ....

Exercise	Points
1	____ 20
2	____ 20
3	____ 20
4	____ 20
5	____ 20
Total	____ 100

Exercise 1.

(i) Find a basis and the dimension of the subspace

$$S = \{(x, y, z, t) \in \mathbb{R}^4 \mid 2x - y = 0 \text{ and } 3z - t = 0\}$$

of $\mathbb{R}^4$.

(ii) Let $E = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = xy\}$. Determine whether E is a subspace of $\mathbb{R}^2$.

Exercise 2. Consider the nonhomogeneous system given by:

$$(*)\quad \begin{cases} x_1 & + & 2x_2 & + & 3x_3 & = 1 \\ 4x_1 & + & 5x_2 & + & 6x_3 & = 1 \\ 7x_1 & + & 8x_2 & + & 9x_3 & = 1 \end{cases}$$

(**a**) Write the matrix form of the system.

(**b**) Find the reduced echelon form of the augmented matrix of the system.

(**c**) Solve the system. Does it have a unique solution?

Exercise 3. Let a be a real number. Consider the system $AX = B$, where

$$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{pmatrix}, \; X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \text{ and } B = \begin{pmatrix} 1 \\ 6 \\ 5 \\ a \end{pmatrix}$$

(**i**) Find $\operatorname{rank}(A)$ and $\operatorname{rank}([A \vdots B])$.

(ii) Find the value of a such that the system $AX = B$ is consistent.

Exercise 4. Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{pmatrix}$.

(**i**) Use Gauss-Jordan Elimination to find A^{-1}.

(ii) Use A^{-1} obtained in (**i**) to solve the following system:

$$\begin{cases} 2x_1 - x_2 + x_3 = 4 \\ 3x_1 - x_3 = 8 \\ x_1 + 2x_2 - x_3 = -16 \end{cases}$$

Exercise 5. Let $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix}$.

(**i**) Explain briefly why A is diagonalizable.

(**ii**) Show that

$$\det(\lambda I_3 - A) = (\lambda + 6)(\lambda - 2)(\lambda - 3).$$

(iii) Find an orthogonal matrix P that diagonalizes A and find the matrix $P^{-1}AP$.