King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 280
Final Exam, Semester I, 2013-2014
Net Time Allowed: 180 minutes

Name:————————————————————————————
ID:——————————————Section:——————Serial:——————————

Q#	Marks	Maximum Marks
1		6
2		5
3		8
4		20
5		10
6		10
7		5
8		10
9		15
10		6
11		15
12		10
13		10
14		10
15		10
Total		150

1. Do not do messy work.
2. Calculators and mobile phones are NOT allowed in this exam.
3. Turn off your mobile.

1. Let S denote the set of all solutions of $y = y(x)$ of the homegenous linear differential equation

$$y'' + 5y' + 6y = 0$$

defined in some interval $[a, b]$. Show that S is a subspace of $C[a, b]$.

2. (5.1) Let $x = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad y = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

   (a) Determine the angle between x and y?

   (b) Determinethe distance between x and y.

3. Find the equation of the plane that passes through the points $P_1 = (2, 3, 1)$, $P_2 = (5, 4, 3)$, and $P_3 = (3, 4, 4)$

4. 5.2 Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 3 & 6 & 1 & 9 & 6 \\ 2 & 4 & 1 & 7 & 5 \end{bmatrix}$$

(a) Find basis and the dimension for $N(A),\ R(A^T), N(A^T)$ and $R(A)$.

(b) Find basis for $N(A)^{\perp},\ R(A^T)^{\perp}, N(A^T)^{\perp}$ and $R(A)^{\perp}$.

5. 5.4 Given

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 1 & 1 \\ -3 & 3 & 2 \\ 1 & -2 & -2 \end{bmatrix}$$

determine the value of each of the following

(a) $\langle A, B \rangle$

(b) $\|A\|_F$

(c) $\|A + B\|_F$

6. Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in \mathbb{R}^2$, define

$$\langle x, y \rangle = 2x_1y_1 - x_1y_2 - x_2y_1 + 5x_2y_2.$$

Prove it is an inner product space.

7. Show that

$$\left| \int_a^b f(x)g(x)dx \right| \leq \left[\int_a^b f^2(x)dx \right]^{1/2} \left[\int_a^b g^2(x)dx \right]^{1/2}$$

8. In the inner product space $P_3(\mathbb{R})$ with

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx.$$

Find the orthogonal complement of the subspace S generated by 1 and x

9. Given the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$

(a) Diagonalize A.

(b) Use the result obtained in (a) to find A^{99}.

10. If A is an invertible matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$. Show that the eigenvalues of A^{-1} are $\lambda_1^{-1}, \cdots, \lambda_n^{-1}$.

11. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 6 \end{bmatrix}$

(a) Find orthonormal basis for the column space S of the matrix A.

(b) Find the QR-factorization of A by using part(a).

12. Let $u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and let L be a linear opreator on $\mathbb{R}^2$ whose matrix representation with respect to the ordered basis $\{u_1, u_2\}$ is

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

(a) Determine the transition matrix from the basis $\{v_1, v_2\}$ to the basis $\{u_1, u_2\}$.

(b) Find the matrix representation of L with respect to $\{v_1, v_2\}$.

13. Given the quadratic equation

$$x^2 + 4xy + y^2 + 3xy + y - 1 = 0$$

find a change of cordinates so that the resulting equation represents a conic in standard position.

14. The function $f(x, y) = (x^2 - 2x)\cos y$ has a critical point at $(1, \pi)$. Determine wether the given stationary point is local maximum, minimum or saddle point.

15. Find bases for the kernel and image of the linear transformation

$$L : \mathbb{R}^2 \to \mathbb{R}^2 \text{ where } L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + 3y \\ 4x + 6y \end{bmatrix}$$