King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 280
Major Exam I, Semester I, 2013-2014
Net Time Allowed: 120 minutes

Name:_____
ID:_____ Section:_____ Serial:_____

Q#	Marks	Maximum Marks
1		10
2		8
3		4
4		6
5		16
6		10
7		10
8		8
9		12
Total		84

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Write the following matrix in reduced row echelon form

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 4 & 2 \\ 4 & 2 & 6 & 8 & 10 \\ 6 & 3 & 14 & 35 & 33 \end{bmatrix}$$

2. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 3 & 1 & a^2 - a + 1 & b + 2 \end{array}\right]$$

(a) For what values of a and b will the system have infinitely many solutions?

(b) For what values of a and b will the system be inconsistent?

3. Let A be a skew symmetric matrix; i.e. $A^T = -A$. Show its diagonal entries must all be zero.

4. Let A be $n \times n$ matrix and let x and y be vectors in $\mathbb{R}^n$. Show that if $Ax = Ay$ and $x \neq y$, then A must be singular.

5. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$

(a) Find elementary matrices E_1, E_2, E_3 such that

$$E_3 E_2 E_1 A = U.$$

(b) Using (a) compute the LU factorization of the matrix A

(c) Using (b), solve the system $Ax = b$, where $b = (1, 4, 4)^T$.

6. Using Cramer's rule, find only the value of y for the system

$$\begin{aligned} x + 2y - z &= 4 \\ -2x + 3y - z &= 1 \\ -2x - 3y + z &= -3. \end{aligned}$$

7. Explain why none of the following is a vector space:

   (a) The set of all ordered pairs of real numbers (x, y) with the operations $(x, y) \oplus (x_0, y_0) = (x - x_0, y - y_0)$ and $c \odot (x, y) = (cx, cy)$.

   (b) The set of all 2×2 real matrices with the operations $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \oplus \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$ and $c \odot \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} ca_1 & -ca_2 \\ -ca_3 & ca_4 \end{bmatrix}$

8. Let E_1, E_2 and E_3 be 3×3 elementary matrices of type I, II and III, respectively, and let A be a 3×3 matrix with $\det(A) = 6$. Assume, additionally, that E_2 was formed from I by multiplying its second row by 3. Find $\det(E_1^T E_2^2 E_3^{-1} A)$.

9. Using the adjoint matrix find the inverse of

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$