King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
(Math 260)

Final Exam
Term 131
Sunday, December 29, 2013
Net Time Allowed: 160 minutes

Name:			
ID:			
Section No:		Serial No:	
Instructor's Name:			

(Show all your steps and work)

Question #	**Marks**
1	14
2	14
3	10
4	14
5	14
6	14
7	12
8	13
9	13
10	14
11	13
12	15
Total	**/160**

(1) Verify that the Differential Equation $\left(3x^2 y + 2xy\right)dx + \left(x^3 + x^2 + 2y\right)dy = 0$ is exact, and then solve it.

[14 points]

(2)

(a). Find the inverse of the matrix: [8 points]

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

(b) Use the result of part **(a)** to solve the system of equations. [6 points]

$$\begin{aligned} x \quad + \quad\quad w &= 2 \\ x - y \quad\quad\quad &= 1 \\ z \quad\quad &= 7 \\ z + w &= 0 \end{aligned}$$

(3) Determine whether or not the set W of all vectors (x_1, x_2, x_3, x_4) in $\mathbb{R}^4$ such that $5x_1 + 2x_4 = x_2 - x_3$ and $x_2 - 2x_3 = x_1 + x_4$ is a subspace of $\mathbb{R}^4$. [10 points]

(4) a) Find the general solution of: [8 points]

$$y^{(4)} + 6y'' + 9y = 0 .$$

b) Find a linear homogenous differential equation whose general solution is:

$$y = (c_1 + c_2 x)e^x$$ [6 points]

(5) Find the eigenvalues and associated eigenvectors of the matrix A:

$$A = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

[14 points]

(6) Determine whether or not the matrix A= $\begin{pmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is diagonalizable and if it is, find a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$. [14 points]

(7) Use Cayley-Hamilton theorem to find A^{-1} if

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

[12 points]

(8) Solve the initial value problem $x' = -y,\ y' = 13x + 4y;\ x(0) = 0,\ y(0) = 3$

[13 points]

(9) The vectors $Y_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $Y_2 = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} e^{-2t}$, $Y_3 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} e^{5t}$ are solutions of the system $Y' = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix} Y \;\; on \; (-\infty, \infty)$

a) Use the Wronskian to show that Y_1, Y_2, Y_3 are linearly independent on $(-\infty, \infty)$.

[6 points]

b) Find a particular solution of the system that satisfies the initial condition $Y(0) = \begin{pmatrix} 10 \\ 1 \\ 6 \end{pmatrix}$.

[7 points]

(10) Find a general solution of the system $Y' = AY, \ where \ \ A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & -4 & 0 \end{pmatrix}$ [14 points]

(11) Find the general solution of the system $Y' = \begin{pmatrix} 9 & 4 & 0 \\ -6 & -1 & 0 \\ 6 & 4 & 3 \end{pmatrix} Y.$ [13 points]

(12) Find the general solution of the system $Y' = \begin{pmatrix} 0 & 1 & 2 \\ -5 & -3 & -7 \\ 1 & 0 & 0 \end{pmatrix} Y.$ [15 points]