Tutorial 3

1. Prove that $4^n > n^3$ for every positive integer n.

2. A sequence $\{a_n\}$ is defined recursively by $a_1 = 1$, $a_2 = 2$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Conjecture a formula for a_n and verify that your conjecture is correct.

3. Let H be a nonempty subset of $\mathbf{Z}$. Suppose that the relation R defined on $\mathbf{Z}$ by $a \; R \; b$ if $a - b \in H$ is an equivalence relation. Verify the following

   (a) $0 \in H$.
   (b) If $a \in H$, then $-a \in H$.
   (c) If $a, b \in H$, then $a + b \in H$.

4. Prove that the multiplication in $\mathbf{Z}_n$, $n \geq 2$, defined by $[a][b] = [ab]$ is well-defined.

5.