Tutorial 3

1 Let $n, m \in \mathbf{Z}$. Prove that if $n \equiv 1 \pmod{2}$ and $m \equiv 3 \pmod{4}$, then $n^2 + m \equiv 0 \pmod{4}$.

2 Show that for every integer n, $4 \nmid (n^2 + 1)$

3 Prove for every two real numbers a and b that $ab \leq \sqrt{a^2}\sqrt{b^2}$.

4 Show that there exist two distinct irrational numbers a and b such that a^b is rational.

5 Disprove the statement: There is a real number x such that $x^6 + x^4 + 1 = 2x^2$.