Tutorial 2

1 Consider the open sentences $P(n) : 5n + 3$ is prime. and $Q(n) : 7n + 1$ is prime., both over the domain $\mathbb{N}$

(a) State $P(n) \Rightarrow Q(n)$ in words.

(b) State $P(2) \Rightarrow Q(2)$ in words. Is this statement true or false?

(c) State $P(6) \Rightarrow Q(6)$ in words. Is this statement true or false?

2 For statements P and Q, show that $(\sim Q) \Rightarrow (P \wedge (\sim P))$ and Q are logically equivalent.

3 For which biconditional is its negation the following?
n^3 and $7n + 2$ are odd or n^3 and $7n + 2$ are even.

4 Let $P(x)$ and $Q(x)$ be open sentences where the domain of the variable x is S. Which of the following implies that $(\sim P(x)) \Rightarrow Q(x)$ is false for some $x \in S$?

(a) $P(x) \wedge Q(x)$ is false for all $x \in S$.

(b) $P(x)$ is true for all $x \in S$.

(c) $Q(x)$ is true for all $x \in S$.

(d) $P(x) \vee Q(x)$ is false for some $x \in S$.

(e) $P(x) \wedge (\sim Q(x))$ is false for all $x \in S$.

5 Prove that if a, b and c are odd integers such that $a + b + c = 0$, then $abc < 0$. (You are permitted to use well-known properties of integers here.)

6 Prove that if x is an odd integer, then $9x + 5$ is even.

7 Let $x \in \mathbb{Z}$. Prove that if 2^{2x} is an odd integer, then 2^{-2x} is an odd integer.

8 Let $S = \{1, 5, 9\}$. Prove that if $n \in S$ and $\frac{n^2+n-6}{2}$ is odd, then $\frac{2n^3+3n^2+n}{6}$ is even.