Tutorial 1

1 Let $A = \{n \in \mathbf{Z} : 2 \le |n| < 4\}$, $B = \{x \in \mathbf{Q} : 2 < x \le 4\}$, $C = \{x \in \mathbf{R} : x^2 - (2+\sqrt{2})x + 2\sqrt{2} = 0\}$ and $D = \{x \in \mathbf{Q} : x^2 - (2+\sqrt{2})x + 2\sqrt{2} = 0\}$.

(a) Describe the set A by listing its elements.
(b) Give an example of three elements that belong to B but do not belong to A.
(c) Describe the set C by listing its elements.
(d) Describe the set D in another manner.
(e) Determine the cardinality of each of the sets A, C and D.

2 For $A = \{x : x = 0 \text{ or } x \in \mathcal{P}(\{0\})\}$, determine $\mathcal{P}(A)$.

3 Give an example of two subsets A and B of $\{1, 2, 3\}$ such that all of the following sets are different: $A \cup B$, $A \cup \overline{B}$, $\overline{A} \cup B$, $\overline{A} \cup \overline{B}$, $A \cap B$, $A \cap \overline{B}$, $\overline{A} \cap B$, $\overline{A} \cap \overline{B}$.

4 For $r \in \mathbf{R}^+$, let $A_r = \{x \in \mathbf{R} : |x| < r\}$. Determine $\bigcup_{r \in \mathbf{R}^+} A_r$ and $\bigcap_{r \in \mathbf{R}^+} A_r$.

5 Give an example of three sets A, S_1 and S_2 such that S_1 is a partition of A, S_2 is a partition of S_1 and $|S_2| < |S_1| < |A|$.

6 For $A = \{a \in \mathbf{R} : |a| \le 1\}$ and $B = \{b \in \mathbf{R} : |b| = 1\}$, give a geometric description of the points in the xy-plane belonging to $(A \times B) \cup (B \times A)$.

7 Let I denote the interval $[0, \infty)$. For each $r \in I$, define

$$A_r = \{(x, y) \in \mathbf{R} \times \mathbf{R} : x^2 + y^2 = r^2\}$$
$$B_r = \{(x, y) \in \mathbf{R} \times \mathbf{R} : x^2 + y^2 \le r^2\}$$
$$C_r = \{(x, y) \in \mathbf{R} \times \mathbf{R} : x^2 + y^2 > r^2\}.$$

(a) Determine $\bigcup_{r \in I} A_r$ and $\bigcap_{r \in I} A_r$.
(b) Determine $\bigcup_{r \in I} B_r$ and $\bigcap_{r \in I} B_r$.
(c) Determine $\bigcup_{r \in I} C_r$ and $\bigcap_{r \in I} C_r$.

8 Give an example of a set $A = \{1, 2, \ldots, k\}$ for a smallest $k \in \mathbf{N}$ containing subsets A_1, A_2, A_3 such that $|A_i - A_j| = |A_j - A_i| = |i - j|$ for every two integers i and j with $1 \le i < j \le 3$.

9 For the open sentence $P(A) : A \subseteq \{1, 2, 3\}$ over the domain $S = \mathcal{P}(\{1, 2, 4\})$, determine:

(a) all $A \in S$ for which $P(A)$ is true.
(b) all $A \in S$ for which $P(A)$ is false.
(c) all $A \in S$ for which $A \cap \{1, 2, 3\} = \emptyset$.

10 State the negation of each of the following statements.

(a) At least two of my library books are overdue.
(b) One of my two friends misplaced his homework assignment.
(c) No one expected that to happen.
(d) It's not often that my instructor teaches that course.
(e) It's surprising that two students received the same exam score.