KFUPM, Department of Mathematics and Statistics
Term 131, Math 232, Final Exam
Date: Thursday January 2, 2014
Time Allowed: 180 minutes

Name: ______________________

ID: ______________________

Section: ______________________

Serial #: ______________________

Question	Points	Out of
1		20
2		20
3		20
4		20
5		20
6		25
7		25
8		25
9		25
Total		200-+

1) Answer in the space provided. You may use the back of the page if necessary.
2) Write clearly and neatly.
3) Please show your work or explain your solutions.
4) No credit will be given for incorrect steps nor will credit be given for correct solutions arrived at which by incorrect means.

Question 1: Let P, Q and R be statements. Without constructing a truth table, show that $\sim(P \Leftrightarrow Q) \equiv \left[(P \vee Q) \wedge \sim(P \wedge Q)\right]$

Question 2: Let A be a nonempty set and let B be a fixed subset of A. A relation R is defined on $\mathcal{P}(A)$ by $X \; R \; Y$ if $X \cap B = Y \cap B$.

(a) Prove that R is an equivalence relation.

(b) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 4\}$. For $X = \{2, 3, 4\}$, determine $[X]$.

Question 3: For a function $f: A \to B$ and subsets C and D of A and E and F of B, prove that

(a) $f(C \cup D) = f(C) \cup f(D)$

(b) $f^{-1}(E - F) = f^{-1}(E) - f^{-1}(F)$.

Question 4: Show that the interval $(-1,1)$ is uncountable.

[Hint: Consider the function $f:(-1,1)\to\mathbf{R}$ defined by $f(x)=\dfrac{x}{1-x^2}$]

Question 5: Let A be a set. Prove that $|A| < |\mathcal{P}(A)|$.

Question 6:

(a) Use the Euclidean Algorithm to find the greatest common divisor for the two integers 1425 and 1994.

(b) Determine integers x and y such that $\gcd(1425, 1994) = 1425x + 1994y$

Question 7:

(a) Let $a, b, c \in \mathbf{Z}$, where a and b are relatively prime nonzero integers. If $a \mid c$ and $b \mid c$, then $ab \mid c$.

(b) Let $a, b, m, n \in \mathbf{Z}$, where $n, m \geq 2$. If $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$, where $\gcd(m, n) = 1$, then $a \equiv b \pmod{mn}$.

Question 8:

(a) Prove that $\lim_{n\to\infty} \frac{3n}{6n+4} = \frac{1}{2}$.

(b) Prove that the infinite series $\sum_{n=1}^{\infty} \frac{3}{9n^2+3n-2}$ converges and find its sum.

Question 9: Give an $\epsilon - \delta$ proof that

(a) $\lim_{x \to 4} \frac{x^2 - 2x - 8}{x^2 - 10x + 24} = -3$,

(b) If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then $\lim_{x \to a} f(x) \cdot g(x) = LM$.