KFUPM, Department of Mathematics and Statistics
Term 131, Math 232, Exam 2
Date: Monday December 9, 2013
Time Allowed: 120 minutes

Name: ______________________________

ID: ______________________________

Section: ______________________________

Serial #: ______________________________

Please Show your work

Question	Points	Out of
1		16
2		12
3		12
4		16
5		16
6		16
7		12
Total		100

Question 1: State and prove The Principle of Mathematical Induction.

Question 2: Use the Principle of Mathematical Induction to prove that for every positive integer n,

$$1\cdot 3\cdot 5\cdots(2n-1)=\frac{(2n)!}{2^n\cdot n!}$$

Question 3: Use the Strong Principle of Mathematical Induction to prove that for each integer $n \geq 12$, there are nonnegative integers a and b such that $n = 3a + 7b$.

Question 4: Consider the subset $H = \{[3k]: k \in \mathbf{Z}\}$ of $\mathbf{Z}_{12}$.

(a) Determine the distinct elements of H and construct an addition table for H.

(b) A relation R on $\mathbf{Z}_{12}$ is defined by $[a]R[b]$ if $[a-b] \in H$. Show that R is an equivalence relation and determine the distinct equivalence classes.

Question 5: The function $f:\mathbf{R}\to\mathbf{R}$ is defined by

$$f(x)=\begin{cases}(x-1)^2 & \text{if } x\le 1\\ \dfrac{1}{1-x} & \text{if } x>1.\end{cases}$$

(a) Show that f is a bijection.

(b) Determine the inverse f^{-1} of f.

Question 6: For nonempty sets A and B and $f : A \to B$ and $g : B \to A$, suspect that $g \circ f = i_A$

(a) Prove that f is one-to-one and g is onto.

(b) Prove that if f is onto, then g is one-to-one.

Question 7: Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \end{pmatrix}, \ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$ be permutations $\mathcal{S}_n$.

Find $x \in \mathcal{S}_n$ such that $x \circ \alpha = \beta$.