**KFUPM, Department of Mathematics and Statistics**
**Term 131, Math 232, Exam 1**
**Date: Saturday October 26, 2013** **Time Allowed: 120 minutes**

Name: ____________________

ID: ____________________

Section: ____________________

Serial #: ____________________

| Question | Points | Out of |
|---|---|---|
| 1 | | 12 |
| 2 | | 10 |
| 3 | | 12 |
| 4 | | 12 |
| 5 | | 12 |
| 6 | | 16 |
| 7 | | 14 |
| 8 | | 12 |
| Total | | 100 |

**Please Show your work**

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**Question 1:** Mark the correct sentence as (**TRUE**) and the incorrect sentence as (**FALSE**).

(a) The set $S=\{X_i : X_i=\{(i,i^2),(i,(i-1)^2)\}, i=1,2\}$ is **NOT** a partition of the set $A\times B$ where $A=\{-1,2\}$ and $B=\{1,4\}$.

(b) Let $A$ and $B$ be two sets such that $A\subset B$. Then there is **NO** set in $\mathcal{P}(B)$ that is disjoint from $A$.

(c) $\left[(P\wedge Q)\Rightarrow(\sim R)\right]\equiv(\sim P)\vee(\sim Q)\vee(\sim R)$ (You can do this without a truth table)

(d) Let $x,y\in\mathbf{Z}$. Then the contrapositive of the implication: "If $(x^2+1)y$ is even, then $x$ is odd or $y$ is even" is "If $x$ is even or $y$ is odd, then $(x^2+1)y$ is odd".

(e) If every month consists of 40 days, then every week consists of 7 days.

(f) Let $S$ be some domain. To prove the statement $\forall\, x\in S,\, P(x)\Rightarrow Q(x)$ by contradiction we prove that $\exists x\in S,\ \sim[(\sim Q(x))\Rightarrow(\sim P(x))]$.

**Question 2:**

(a) If $\bigcap_{\alpha \in I} A_\alpha = \bigcup_{\alpha \in I} A_\alpha$ for some indexed collection of sets $\{A_\alpha\}_{\alpha \in I}$ with and indexed set $I$ . What can you say about the relationship between the sets $A_\alpha$ .

(b) Let $n \in \mathbf{N}$, and define the sets $A_n = \{0, \pm 1, \pm 2, \cdots, \pm n\}$. Then

(i) $\bigcap_{i \in \mathbf{N}} A_i =$

(ii) $\bigcup_{i \in \mathbf{N}} A_i =$

**Question 3:** Show whether the following statement is a tautology or not.

$$[(P \wedge R) \Rightarrow Q] \Leftrightarrow [P \Rightarrow (R \Rightarrow Q)]$$

**Question 4:**

(a) Each of the following describes an implication. Write the implication in the form “if, then.”

(i) Any point on the straight line with equation $2y + x - 3 = 0$ whose $x -$ coordinate is an integer also has an integer for its $y -$ coordinate.

(ii) Let $n \in \mathbf{Z}$. Whenever $3n + 7$ is even, $n$ is odd.

(iii) For an integer to be odd, it is sufficient that its square be odd.

(iv) A matrix $A$ is invertible only if $\det A \neq 0$.

(b) For the open sentences $P(x): |x + 1| < 2$ and $Q(x): x \in (-3,1)$ over the domain $\mathbf{R}$. State the biconditional $P(x) \Leftrightarrow Q(x)$ in two ways: one using “if and only if” and the other using “necessary and sufficient”.

**Question 5:** Let $A$ and $B$ be two sets. Then

(a) Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

(b) Disprove that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

**Question 6:**

(a) For an integer $x$, prove that $3 | x^2$ if and only if $3 | x$.

(b) Prove that $\sqrt{3}$ is an irrational number.

**Question 7:** Let $x$ be a positive real number. Prove that if $x - \frac{2}{x} > 1$, then $x > 2$ by

(a) a proof by contrapositive.

(b) a proof by contradiction

**Question 8:**

(a) Prove that there is no integer $n$ such that $n \equiv 5(\mathrm{mod}\,14)$ and $n \equiv 3(\mathrm{mod}\,21)$.

(b) Evaluate the proposed proof of the following result.

**<u>Result</u>**:

If $x$ is an irrational number and $y$ is a rational number, then $z = x - y$ is irrational.

**<u>Proof</u>**:

Assume, to the contrary, that $z = x - y$ is rational. Then $z = \frac{a}{b}$, where $a,b \in \mathbf{Z}$ and $b \neq 0$. Since $\sqrt{3}$ is irrational, we let $x = \sqrt{3}$. Since $y$ is rational, $y = \frac{c}{d}$, for some integers $c,d$ with $d \neq 0$. Therefore,

$$\sqrt{3} = x = y + z = \frac{c}{d} + \frac{a}{b} = \frac{ad + bc}{bd}.$$

Since $ad + bc$ and $bd$ are integers, where $db \neq 0$, it follows that $\sqrt{3}$ is rational, producing a contradiction.