Name:.. ID#:................................. Serial#:...........

1. Find an interval centered about $x = 0$ for which the initial-value problem
   $(x - 1) y'' + (\sec x) y = e^x$, $y(0) = 0, y'(0) = 1$ has a unique solution.

2. Verify that $y = c_1 \cos(2x) + c_2 \sin(2x) + \frac{3}{4}x^3 - \frac{9}{8}x$ is the general solution of the non-homogenous differential equation $y'' + 4y = 3x^3$ on the interval $(-\infty, \infty)$.

3. Determine whether the functions $f_1(x) = 3\cos^2 x$, $f_2(x) = 2\sin^2 x$, $f_3(x) = 17$ is linearly independent on the interval $(-\infty, \infty)$.

4. Find a second solution $y_2(x)$ of the differential equation

$$(1 - x^2)y'' + 2xy' - 2y = 0$$

where the first solution is $y_1(x) = x$.

5. Given that $y = c_1 e^t \sin 2t + c_2 e^t \cos 2t$ is a two-parameter family of solutions of the differential equation

$$y'' - 2y' + 5y = 0$$

Determine whether a member of the family of the solutions of the above differential equation can be found that satisfies the boundary conditions: $y(0) = 1, y(\pi) = -1$

6. Given that $y = x^2 + 8x + 33$ is a particular solution to the differential equation

$$y'' + 4y' - y = -x^2 - 1$$

And $y = -\frac{1}{4}xe^{-x} + \frac{1}{8}e^{-x}$ is a particular solution to the differential equation

$$y'' + 4y' - y = xe^{-x} - e^{-x}$$

Find a particular solution to the differential equation

$$y'' + 4y' - y = -x^2 - 1 - 2xe^{-x} + 2e^{-x}$$

7. Solve the initial value problem

$$y^{(4)} = y''' + y'' + y' + 2y; \qquad y(0) = y'(0) = y''(0) = 0, y'''(0) = 30$$