MATH 202.6 (Term 131)

Quiz 5 (Sects. 6.1, 6.2) Duration: 20mn

Name: ID number:

Find two linearly independent power series solutions of $y'' + xy' + 3y = 0$, about the ordinary point $x = 0$. Give the first four terms of the series solutions.

$$y = \sum_{n=0}^{\infty} c_n x^n \Rightarrow y' = \sum_{n=1}^{\infty} c_n n x^{n-1} \Rightarrow y'' = \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2}$$

So, we have

$$\sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} + x \sum_{n=1}^{\infty} c_n n x^{n-1} + 3 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} + \sum_{n=1}^{\infty} c_n n x^n + 3 \sum_{n=0}^{\infty} c_n x^n = 0$$

$n-2=k$ $\Downarrow$

$$\Rightarrow \sum_{k=0}^{\infty} c_{k+2}(k+1)(k+2) x^k + \sum_{k=1}^{\infty} c_k k x^k + 3 \sum_{k=0}^{\infty} c_k x^k = 0$$

$$2c_2 + 3c_0 + \sum_{k=1}^{\infty} \left[c_{k+2}(k+1)(k+2) + c_k k + 3c_k \right] x^k = 0$$

$$\begin{cases} 2c_2 + 3c_0 = 0 \\ c_{k+2}(k+1)(k+2) + c_k(k+3) = 0, \quad k = 1, 2, \ldots \end{cases}$$

$$c_2 = -\frac{3}{2} c_0, \qquad c_{k+2} = -\frac{(k+3)}{(k+1)(k+2)} c_k, \quad k = 1, 2, \ldots$$

$$c_3 = -\frac{4}{2\cdot 3} c_1 = -\frac{2}{3} c_1; \quad c_4 = -\frac{5}{3\cdot 4} c_2 = \frac{5\cdot 3}{3\cdot 4\cdot 2} c_0 = \frac{5c_0}{8}$$

$$c_5 = -\frac{6}{4\cdot 5} c_3 = \frac{6}{4\cdot 5}\frac{2}{3} c_1 = \frac{c_1}{5}; \quad c_6 = -\frac{7}{5\cdot 6} c_4 = -\frac{7}{5\cdot 6}\frac{5}{8} c_0 = -\frac{7}{48} c_0$$

$$c_7 = -\frac{8}{6\cdot 7} c_5 = -\frac{8}{6\cdot 7}\frac{c_1}{5} = -\frac{4}{105} c_1$$

$$\Rightarrow y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 + \cdots$$

$$= c_0 + c_1 x - \frac{3}{2} c_0 x^2 - \frac{2}{3} c_1 x^3 + \frac{5}{8} c_0 x^4 + \frac{c_1}{5} x^5 - \frac{7}{48} c_0 x^6 - \frac{4}{105} c_1 x^7 + \cdots$$

$$= c_0 \underbrace{\left(1 - \frac{3}{2}x^2 + \frac{5}{8}x^4 - \frac{7}{48}x^6 + \cdots\right)}_{y_1} + c_1 \underbrace{\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5 - \frac{4}{105}x^7 + \cdots\right)}_{y_2}$$

MATH 202.9 (Term 131)

Quiz 5 (Sects. 6.1, 6.2) Duration: 20mn

Name:

ID number:

Find two linearly independent power series solutions of $y'' - 2xy = 0$, about the ordinary point $x = 0$. Give the first four terms of the series solutions.

$$y = \sum_{n=0}^{\infty} c_n x^n \Rightarrow y' = \sum_{n=1}^{\infty} c_n n x^{n-1} \Rightarrow y'' = \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2}.$$

$$\Rightarrow \sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} - 2x \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} - 2 \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$n-2=k$ $\qquad$ $n+1=k$

$$\sum_{k=0}^{\infty} c_{k+2}(k+1)(k+2) x^k - 2 \sum_{k=1}^{\infty} c_{k-1} x^k = 0$$

$$2c_2 + \sum_{k=1}^{\infty} \left[c_{k+2}(k+1)(k+2) - 2c_{k-1} \right] x^k = 0$$

$2c_2 = 0$, $\quad c_{k+2}(k+1)(k+2) - 2c_{k-1} = 0$, $k = 1, 2, \ldots$

$c_2 = 0$, $\quad c_{k+2} = \dfrac{2}{(k+1)(k+2)} c_{k-1}$, $k = 1, 2, \ldots$

$c_3 = \dfrac{2}{2\cdot 3} c_0 = \dfrac{c_0}{3}$; $\quad c_4 = \dfrac{2}{3\cdot 4} c_1 = \dfrac{1}{6} c_1$, $\quad c_5 = 0$

$c_6 = \dfrac{2}{5\cdot 6} c_3 = \dfrac{c_0}{45}$; $\quad c_7 = \dfrac{2}{6\cdot 7} c_4 = \dfrac{c_1}{126}$, $\quad c_8 = 0$

$$\Rightarrow y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 + c_8 x^8 + \ldots$$

$$= c_0 + c_1 x + \frac{c_0}{3} x^3 + \frac{c_1}{6} x^4 + \frac{c_0}{45} x^6 + \frac{c_1}{126} x^7 + \ldots$$

$$= c_0 \underbrace{\left(1 + \frac{x^3}{3} + \frac{x^6}{45} + \ldots\right)}_{y_1} + c_1 \underbrace{\left(x + \frac{x^4}{6} + \frac{x^7}{126} + \ldots\right)}_{y_2}$$