MATH 202.6 (Term 131)

Quiz 4 (Sects. 4.5, 4.6) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the ODE $y'' - 4y = 4e^{5x} + 7\cos 2x$ by using annihilator approach.

2.)(5pts)Solve the ODE $y'' + 3y' + 2y = \frac{1}{e^x+1}$ by using variations of parameters.

1.) $m^2-4=0$, $m=\pm 2$

$\Rightarrow y_c = c_1 e^{-2x} + c_2 e^{2x}$

$(D-5)(D^2+4)\left[4e^{5x}+7\cos 2x\right]=0$

$(D^2-4)(D-5)(D^2+4)\,y=0$

$m^2=4$, $m=5$, $m=\pm 2i$

$\Rightarrow y = \underbrace{c_1 e^{-2x}+c_2 e^{2x}}_{y_c} + \underbrace{c_3 e^{5x}+c_4\cos 2x + c_5\sin 2x}_{y_p}$

The form of y_p is

$y_p = Ae^{5x} + B\cos 2x + C\sin 2x$

$y_p' = 5Ae^{5x} - 2B\sin 2x + 2C\cos 2x$

$y_p'' = 25Ae^{5x} - 4B\cos 2x - 4C\sin 2x$

$y_p'' - 4y_p = 4e^{5x} + 7\cos 2x \Rightarrow$

$25Ae^{5x} - 4B\cos 2x - 4C\sin 2x$
$-4Ae^{5x} - 4B\cos 2x - 4C\sin 2x =$
$4e^{5x} + 7\cos 2x$

$\Rightarrow 21A = 4 \longrightarrow A = 4/21$

$-8B = 7 \longrightarrow B = -7/8$

$-8C = 0 \longrightarrow C = 0$

$$\boxed{y = c_1 e^{-2x} + c_2 e^{2x} + \frac{4}{21}e^{5x} - \frac{7}{8}\cos 2x}$$

2.) $m^2+3m+2=0$, $D = 9-8 = 1$

$m_1 = \frac{-3-1}{2} = -2$, $m_2 = \frac{-3+1}{2} = -1$

$\Rightarrow y_c = c_1 \underbrace{e^{-x}}_{y_1} + c_2 \underbrace{e^{-2x}}_{y_2}$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$

$u_1' = \frac{-y_2 f(x)}{W} = +\frac{e^{-2x}}{e^{-3x}}\left(\frac{1}{e^x+1}\right) = \frac{e^x}{e^x+1}$

$u_1 = \int \frac{e^x}{e^x+1}\,dx = \ln(e^x+1)$

$u_2' = \frac{y_1 f(x)}{W} = -\frac{e^{-x}}{e^{-3x}}\left(\frac{1}{e^x+1}\right) = -\frac{e^{2x}}{e^x+1}$

$u_2 = -\int \frac{e^{2x}}{e^x+1}\,dx$

$v = e^x$, $dv = e^x\,dx$

$u_2 = -\int \frac{v}{v+1}\,dv = -\int \left(1 - \frac{1}{v+1}\right)dv$

$= -\left[v - \ln(v+1)\right] = -\left[e^x - \ln(e^x+1)\right]$

$\Rightarrow y_p = e^{-x}\ln(e^x+1) - e^{-2x}\left[e^x - \ln(e^x+1)\right]$

$= -e^{-x} + (e^{-x} + e^{-2x})\ln(e^x+1)$

$$\Rightarrow \boxed{y = c_1 e^{-x} + c_2 e^{-2x} - e^{-x} + (e^{-x}+e^{-2x})\ln(e^x+1)}$$

MATH 202.9 (Term 131)

Quiz 4 (Sects. 4.5, 4.6) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the DE $y''+2y'-3y=-2e^{-4x}+8\sin 3x$ **by using annihilator approach.**

2.)(5pts)Solve the DE $y''-y=\frac{2e^{x}}{e^{x}+e^{-x}}$ **by using variations of parameters.**

1.) $m^2+2m-3=0$ $\quad \Delta = 4+12=16$

$m_1=\frac{-2-4}{2}=-3, \quad m=\frac{-2+4}{2}=1$

$\Rightarrow y_c = c_1e^{-3x}+c_2e^{x}.$

Now, $(D+4)(D^2+9)\left[-2e^{-4x}+8\sin 3x\right]=0$

$\Rightarrow (D^2+2D-3)(D+4)(D^2+9)(y)=0$

$m^2+2m-3=0 \quad m=-4, \quad m=\pm 3i$

$\Rightarrow y=\underbrace{c_1e^{-3x}+c_2e^{x}}_{y_c}+\underbrace{c_3e^{-4x}+c_4\cos 3x+c_5\sin 3x}_{y_p}$

The form of y_p is

$y_p = Ae^{-4x}+B\cos 3x + C\sin 3x$

$y_p' = -4Ae^{-4x}-3B\sin 3x+3C\cos 3x$

$y_p'' = 16Ae^{-4x}-9B\cos 3x-9C\sin 3x$

$y_p''+2y_p'-3y_p=-2e^{-4x}+8\sin 3x$

$\Rightarrow 5Ae^{-4x}+(6C-12B)\cos 3x-(6B+12C)\sin 3x=-2e^{-4x}+8\sin 3x$

$\Rightarrow 5A=-2 \quad \rightarrow \quad A=-2/5$

$\left.\begin{array}{l}6C-12B=0\\ 6B+12C=-8\end{array}\right\}$ $\begin{array}{l}C=2B\\ 6B+24B=-8\end{array}$

$B=-8/30=-4/15$

$C=-8/15$

$\Rightarrow \boxed{y=c_1e^{-3x}+c_2e^{x}-\frac{2}{5}e^{-4x}-\frac{4}{15}\cos 3x-\frac{8}{15}\sin 3x.}$

2.) $m^2-1=0, \quad m=\pm 1$

$\Rightarrow y_c = c_1\underbrace{e^{-x}}_{y_1}+c_2\underbrace{e^{x}}_{y_2}.$

$W=\begin{vmatrix} e^{-x} & e^{x} \\ -e^{-x} & e^{x}\end{vmatrix}=2$

$u_1'=\frac{-y_2 f(x)}{W}=-\frac{e^{x}}{2}\left(2\frac{e^{x}}{e^{x}+e^{-x}}\right)=-\frac{e^{2x}}{e^{x}+e^{-x}}$

$u_1'=-\frac{e^{3x}}{e^{2x}+1} \Rightarrow u_1=-\int\frac{e^{3x}}{e^{2x}+1}dx$

If $v=e^{x}$, $dv=e^{x}dx$

$u_1=-\int\frac{v^2}{v^2+1}dv=-\int\left(1-\frac{1}{v^2+1}\right)dv$

$=-\left[v-\tan^{-1}v\right]$

$u_1(x)=-\left[e^{x}-\tan^{-1}e^{x}\right]$

$u_2'=\frac{y_1 f(x)}{W}=\frac{e^{-x}}{2}\left(2\frac{e^{x}}{e^{x}+e^{-x}}\right)=\frac{1}{e^{x}+e^{-x}}$

$u_2'=\frac{e^{x}}{e^{2x}+1)} \quad u_2=\int\frac{e^{x}}{e^{2x}+1}dx$

$v=e^{x}, \quad dv=e^{x}dx$

$u_2=\int\frac{dv}{v^2+1}=\tan^{-1}v$

$=\tan^{-1}(e^{x})$

$\Rightarrow y_p=-e^{-x}\left[e^{x}-\tan^{-1}e^{x}\right]+e^{x}\tan^{-1}e^{x}$

$=-1+(e^{x}+e^{-x})\tan^{-1}e^{x}$

$\boxed{y=c_1e^{-x}+c_2e^{x}-1+(e^{-x}+e^{x})\tan^{-1}e^{x}}.$