MATH 202.9 (Term 131)

Quiz 3 (Sects. 4.2, 4.3) Duration: 20mn

Name: ID number:

1.) a.)(5pts)Use reduction of order to find a second solution of the DE y_2 of $(1-2x-x^2)y''+2(1+x)y'-2y=0$, given that $y_1=x+1$ is a solution of the DE.

2.)(5pts)Solve the IVP $\begin{cases} y'''+y''+y'+y=0 \\ y(0)=y'(0)=0,\ y''(0)=2. \end{cases}$

---

1) $y''+\dfrac{2(1+x)}{1-2x-x^2}y'-\dfrac{2}{1-2x-x^2}y=0$

$P(x)=\dfrac{2(1+x)}{1-2x-x^2}$

$y_2=y_1(x)\displaystyle\int\frac{e^{-\int P(x)dx}}{y_1^2(x)}$

$=(x+1)\displaystyle\int\frac{e^{-\int\frac{2(1+x)}{1-2x-x^2}dx}}{(x+1)^2}dx$

$=(x+1)\displaystyle\int\frac{e^{\ln|1-2x-x^2|}}{(x+1)^2}dx$

$=(x+1)\displaystyle\int\frac{1-2x-x^2}{(x+1)^2}dx,\quad 1-2x-x^2>0$

but, $1-2x-x^2=2-(1+x)^2$

Thus, $y_2(x)=(x+1)\displaystyle\int\left(\frac{2}{(x+1)^2}-1\right)dx$

$=(x+1)\left[-\dfrac{2}{x+1}-x\right]$

$=-2-x(x+1)$

2) The auxiliary equation is

$m^3+m^2+m+1=0$

$m^2(m+1)+(m+1)=0$

$(m+1)(m^2+1)=0$

$m=-1,\quad m=\pm i$

$y=c_1e^{-x}+c_2\cos x+c_3\sin x$

$y'=-c_1e^{-x}-c_2\sin x+c_3\cos x$

$y''=c_1e^{-x}-c_2\cos x-c_3\sin x$

$\begin{array}{ll} y(0)=0 \Rightarrow & c_1+c_2=0 \quad (1) \\ y'(0)=0 \Rightarrow & -c_1+c_3=0 \quad (2) \\ y''(0)=2 \Rightarrow & c_1-c_2=2 \quad (3) \end{array}$

$(1)\Rightarrow c_2=-c_1$

$(3)\Rightarrow c_1+c_1=2\Rightarrow c_1=1$

$c_2=-1$

$(2)\Rightarrow c_3=c_1=1$

$\Rightarrow y=e^{-x}-\cos x+\sin x$

MATH 202.6 (Term 131)

Quiz 3 (Sects. 4.2, 4.3) Duration: 20mn

Name: ID number:

1.) a.)(2pts)Find m such that e^{mx} is solution of $xy'' - (x+1)y' + y = 0$.
b.)(3pts)Find a second solution by reduction of order.

2.)(5pts)Solve the IVP $\begin{cases} y^{(4)} - 7y'' - 18y = 0 \\ y(0) = y'(0) = 0,\ y''(0) = y'''(0) = 1 \end{cases}$

---

1) $y = e^{mx}$

a) We substitute y into the DE.

$xm^2 e^{mx} - (x+1) m e^{mx} + e^{mx} = 0$

$[xm^2 - (x+1)m + 1] e^{mx} = 0$

$\Rightarrow xm^2 - (x+1)m + 1 = 0$

$(m^2 - m)x - m + 1 = 0$

This implies $\begin{cases} m^2 - m = 0 \\ -m + 1 = 0 \end{cases}$

$m = 1$

$\Rightarrow y_1 = e^x$ is a solution.

b) $y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y_1^2(x)}\,dx$

$= e^x \int \frac{e^{\int \frac{x+1}{x}dx}}{e^{2x}}\,dx = e^x \int \frac{e^{x+\ln x}}{e^{2x}}\,dx$

$= e^x \int \frac{x e^x}{e^{2x}}\,dx, \quad x>0$

$= e^x \int x e^{-x}\,dx = e^x\left(-(x+1)e^{-x}\right)$

$y_2 = -(x+1), \quad x>0$

2.) The auxiliary equation is

$m^4 - 7m^2 - 18 = 0$

Let $m^2 = \alpha$,

$\alpha^2 - 7\alpha - 18 = 0$

$\Delta = 49 + 4\cdot 18 = 121 = 11^2$

$\alpha_1 = \frac{7-11}{2} = -2$

$\alpha_2 = \frac{7+11}{2} = 9$

So that, $m^2 = -2 \Rightarrow m = \pm i\sqrt{2}$

$m^2 = 9 \Rightarrow m = \pm 3$

$y = c_1 e^{-3x} + c_2 e^{3x} + c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x$

$y' = -3c_1 e^{-3x} + 3c_2 e^{3x} - \sqrt{2}c_3 \sin\sqrt{2}x + c_4\sqrt{2}\cos\sqrt{2}x$

$y'' = 9c_1 e^{-3x} + 9c_2 e^{3x} - 2c_3 \cos\sqrt{2}x - 2c_4 \sin\sqrt{2}x$

$\begin{array}{ll} y(0) = 0 \Rightarrow & c_1 + c_2 + c_3 = 0 \\ y'(0) = 0 \Rightarrow & -3c_1 + 3c_2 + c_4\sqrt{2} = 0 \\ y''(0) = 1 \Rightarrow & 9c_1 + 9c_2 - 2c_3 = 1 \\ y'''(0) = 1 \Rightarrow & -27c_1 + 27c_2 - 2\sqrt{2}c_4 = 1 \end{array}$

We solve this system to find c_1, c_2, c_3 and c_4.