MATH 202.6 (Term 131)

Quiz 2 (Chap. 2.4-2.5) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the exact DE: $(\sin^2(x+y)+\frac{x^2y^3}{3})dx+(-\frac{1}{2}\cos 2(x+y)+\frac{x^3y^2}{3})dy=0.$

2.)(5pts)Solve by substitution the DE: $\frac{dy}{dx}=\frac{x-y+5}{x-y}$.

Solution

1.) $M=\sin^2(x+y)+x^2y^3/3$

$\Rightarrow M_y = 2\cos(x+y)\sin(x+y)+x^2y^2$

$= \sin 2(x+y) + x^2y^2$

$N=-\frac{1}{2}\cos 2(x+y)+x^3y^2/3$

$\Rightarrow N_x = \sin 2(x+y)+x^2y^2$

$M_y=N_x \Rightarrow$ DE exact.

$\frac{\partial f}{\partial x}=\sin^2(x+y)+\frac{x^2y^3}{3}, \quad \frac{\partial f}{\partial y}=-\frac{1}{2}\cos 2(x+y)+\frac{x^3y^2}{3}$

$f(x,y)=\frac{-\sin 2(x+y)}{4}+\frac{x^3y^3}{9}+h(x)$

$\frac{\partial f}{\partial x}=-\frac{\cos 2(x+y)}{2}+\frac{x^2y^3}{3}+h'(x)=$

$\sin^2(x+y)+\frac{x^2y^3}{3}$

Noting that $\sin^2(x+y)=\frac{1-\cos 2(x+y)}{2}$,

we find $h'(x)=\frac{1}{2}$

$h(x)=\frac{x}{2}+C$

$\Rightarrow \boxed{\frac{-\sin 2(x+y)}{4}+\frac{x^2y^3}{3}+\frac{x}{2}=C}$

2.) $\frac{dy}{dx}=\frac{x-y+5}{x-y}$

$u=x-y \Rightarrow \frac{du}{dx}=1-\frac{dy}{dx}$

We substitute into the DE, and we find

$1-\frac{du}{dx}=\frac{u+5}{u}$

$\frac{du}{dx}=1-\frac{u+5}{u}=-\frac{5}{u}$

$\int u\,du=-\int 5\,dx$

$u^2=-5x+C$

$\boxed{(x-y)^2=-5x+C}$

MATH 202.9 (Term 131)

Quiz 2 (Chap. 2.4-2.5) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the exact DE: $(\cos^2(x+y) + \frac{xy^2}{2})dx + (\frac{1}{2}\cos 2(x+y) + \frac{x^2y}{2})dy = 0$.

2.)(5pts)Solve by substitution the DE: $\frac{dy}{dx} = \frac{\sqrt{-x+y}+2}{\sqrt{-x+y}}$.

Solution

1.) $M = \cos^2(x+y) + xy^2/2$

$N = \frac{1}{2}\cos 2(x+y) + \frac{x^2y}{2}$

$\Rightarrow M_y = -2\cos(x+y)\sin(x+y) + xy$

$= -\sin 2(x+y) + xy$

$\Rightarrow N_x = -\sin 2(x+y) + xy$

$M_y = N_x \Rightarrow$ DE is exact.

$\frac{\partial f}{\partial x} = \cos^2(x+y) + \frac{xy^2}{2}$; $\frac{\partial f}{\partial y} = \frac{1}{2}\cos 2(x+y) + \frac{x^2y}{2}$

$f(x,y) = \frac{\sin 2(x+y)}{4} + \frac{x^2y^2}{4} + h(x)$

$\frac{\partial f}{\partial x} = \frac{\cos 2(x+y)}{2} + \frac{xy^2}{2} + h'(x) =$

$\cos^2(x+y) + \frac{xy^2}{2}$.

Noting that $\cos^2(x+y) = \frac{\cos 2(x+y)+1}{2}$

$\Rightarrow h'(x) = \frac{1}{2}$.

$h(x) = x/2$.

$\Rightarrow \boxed{\frac{\sin 2(x+y)}{4} + \frac{xy^2}{2} + \frac{x}{2} = C}$

2.) $\frac{dy}{dx} = \frac{\sqrt{-x+y}+2}{\sqrt{-x+y}}$

let $u = -x+y$, $\frac{du}{dx} = -1 + \frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} + 1 = \frac{\sqrt{u}+2}{\sqrt{u}}$

$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{u}} \Rightarrow \int \sqrt{u}\,du = 2\int dx$

$\frac{2}{3}u^{3/2} = 2x + C$

$\frac{2}{3}(-x+y)^{3/2} = 2x + C$

$\boxed{(-x+y)^{3/2} = 3x + C}$