1. (6 points)Find the value of c such that $y(x) = c(1 - e^{-20x})$ is a solution of the differential equation
$$y' + 20y = 24.$$

2. (6 points)Determine a region of the xy-plane for which the differential equation
$$(y^3 - 1)\, y' = x$$
would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

3. (4 points) Let $y = 1/(c-x)$ be a one-parameter family of solutions of the differential equation $dy/dx = y^2$.

   a) Determine the value of the parameter in the given solution so that the solution satisfies the initial-value-problem

   $$dy/dx = y^2, \;\; y(1) = 2$$

   b) (2 points) Give the largest interval of definition in which the solution of the initial-value-problem in part (a) lies.

   c) (2 points) Sketch the solution of the initial-value-problem in part (a).

4. a)(4 points) Consider the first order differential equation $(y^2-1)dx+xydy=0$. Determine whether the given DE is linear (i) w.r.t x (ii) w.r.t y

b) (4 points)What is the order of the ODE. Determine whether it is linear or non-linear.

(i) $\left(\dfrac{dy}{dx}\right)^2=\left(\dfrac{d^2y}{dx^2}+y\right)^{3/2}-6x+2y$

(ii) $y''+y\sin x=0$

5. (12 points)Solve the initial-value-problem:

$$e^{-2y}\frac{dx}{dy} = (x - x^2)y,\ x(0) = \frac{1}{2}$$

6. (12 points)Solve the initial-value-problem:

$$\begin{cases} \cos x \dfrac{dy}{dx} + (\sin x)y = \cos^4 x \\ y(0) = 1 \end{cases}$$

7. (12 points)Show that the differential equation is exact and solve it:

$$y' = \frac{2xy - \sin x - 3y^2 e^x}{6ye^x - x^2}$$

8. (12 points)Use an appropriate substitution to transform the given DE into a linear differential equation. (**Do not solve the linear differential equation**)

$$x\frac{dy}{dx} - (1+x)y = xy^2$$

9. (12 points) Solve $(x^3 + y^2\sqrt{x^2+y^2})\,dx - xy\sqrt{x^2+y^2}\,dy = 0\,(x > 0)$.

10. (12 points) An object, initially at $75°F$, is placed in a $475°F$ oven at 5:00 PM. After 75 minutes, it is found that the temperature of the object is $275°F$. When will the object be at $375°F$?