King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 201 - Final Exam - Term 131

Duration: 180 minutes
Code 004

Name:______________________________ ID Number:______________________________

Section Number:____________________ Serial Number:___________________________

Class Time: ________________________ Instructor's Name:______________________

Instructions:

1. Write your name, ID number and Section number on the examination paper on the given answer sheet for the MCQS.
2. Write neatly and eligibly. You may lose points for messy work.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 14 pages of problems, (**7 pages of written questions and 7 pages of MCQS**)
5. The maximum point for each page is 10
6. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
7. Use a good eraser. DO NOT use the erasers attached to the pencil.
8. Calculators and Mobiles are not allowed.

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1. A) A curve $\mathcal{C}$ is defined by the parametric equations

$$x = 1 + \sin t, \qquad y = \cos t + 2, \qquad 0 \le t \le \pi.$$

Find the cartesian equation for $\mathcal{C}$, sketch the curve $\mathcal{C}$ and the direction of motion.

B) Find the area of the region that is inside $r = \cos\theta$ but outside $r = \sin\theta$.

2. A) Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$.

B) If $\vec{u} = \langle 0, 3, 4 \rangle$ and $\vec{v} = \langle 10, 11, -2 \rangle$ then find the scalar component of $\vec{u}$ in the direction of $\vec{v}$.

3. A) Find and sketch the domain of

$$f(x,y) = \frac{1}{\ln(9 - x^2 - y^2)}.$$

B) Find and sketch the level curve of $f(x,y) = \dfrac{1}{\ln(9 - x^2 - y^2)}$ passing through the point $(2,1)$.

4. Find the local extrema of the function $f(x, y) = x^3 + 3xy^2 - 15x + y^3 - 15y$.

5. A flat circular plate has the shape of the region $x^2+y^2 \leq 1$. The plate (including the boundary $x^2+y^2=1$) is heated so that the temperature at any point (x,y) on the plate is given by $T(x,y)=x^2+2y^2-x$. Find the temperatures at the hottest and coldest points on the plate, including the boundary $x^2+y^2=1$.

6. A) Evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 \frac{\sin(x^2)}{x}\, dx\, dy.$$

B) Evaluate

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz\, dx\, dy.$$

7. A) Let $f(x,y) = 2\sqrt{x^2+4y}$. Find the directional derivative of f at $P = (-2,3)$ in the direction starting from P pointing towards $Q = (0,4)$.

B) Find all unit vectors $\vec{u}$ for which $D_{\vec{u}}f(-2,3) = 0$, where f is the function in part A).

8. The arc length of the parametrized curve

$$x = 4\cos t + 4t\sin t, \qquad y = 4\sin t - 4t\cos t, \qquad 0 \le t \le \frac{\pi}{2}.$$

is equal to

a) $\frac{\pi^2}{2}$
b) $\frac{\pi}{4}$
c) $4\,\pi^2$
d) $\pi^2 + 2$
e) $2\,\pi^2$

9. The graph of $x^2 + 2y^2 - 3z^2 + 4x + 3 = 0$ is

a) A hyperboloid of one sheet
b) A hyperboloid of two sheets
c) A cone
d) An ellipsoid
e) A paraboloid

10. The volume of the solid that lies below the graph of $f(x,y) = e^{y+2x}$ and above the rectangle $R = \{(x,y)|\ 0 \le x \le \ln 3,\ \ 0 \le y \le \ln 3\}$ is equal to

a) 8

b) $2\ln 3$

c) $\frac{9}{2}$

d) 5

e) $(\ln 3)^2$

11. Let L_1 and L_2 be the lines given by the parametric equations

$$L_1 : x = 1 + t, \quad y = 2t, \quad z = 3 + t$$

$$L_2 : x = 1 - t, \quad y = 2 - 2t, \quad z = 5 - t.$$

An equation of the plane which contains both lines is given by

a) $x - y + z - 4 = 0$

b) $3x + y + z - 6 = 0$

c) $x + y - z + 2 = 0$

d) $x + 2y + 5z = 0$

e) $x + 2y + z - 5 = 0$

12. The distance from the origin to the line given by

$$x = 2 - t, \quad y = -2t, \quad z = -1 + 3t$$

is equal to

a) $\sqrt{\frac{45}{14}}$

b) $\sqrt{\frac{22}{3}}$

c) $\sqrt{\frac{2}{7}}$

d) $\sqrt{\frac{14}{21}}$

e) $\sqrt{\frac{33}{7}}$

13. The volume of the parallelpiped with vertices $A(0,0,0)$, $B(1,-1,1)$, $C(2,1,-2)$ and $D(-1,2,-1)$ is equal to

a) 4

b) 6

c) 2

d) 5

e) 8

14. If the point $(9, -9)$ is given in rectangular coordinates, then its polar coordinate may be given by

a) $\left(9\sqrt{2}, \frac{7\pi}{4}\right)$
b) $\left(9\sqrt{2}, \frac{\pi}{4}\right)$
c) $\left(9, \frac{7\pi}{4}\right)$
d) $\left(-9, \frac{7\pi}{4}\right)$
e) $\left(\sqrt{2}, \frac{7\pi}{4}\right)$

15. If $\mathcal{R}$ is the region in the first quadrant bounded by the parabolas $x = 8 - y^2$ and $x = y^2$, then $\int\int_{\mathcal{R}} y\, dA$ is equal to

a) 8
b) 0
c) 4
d) $\frac{1}{8}$
e) $\frac{1}{4}$

16. The volume V of the solid under the surface $z = 1 - x^2 - y^2$ and above the xy-plane is equal to

a) $\dfrac{\pi}{2}$

b) $\dfrac{\pi}{4}$

c) $2\,\pi$

d) $\dfrac{1}{2} + \pi$

e) $\dfrac{3\pi}{2}$

17. A normal vector to the tangent plane of the level surface

$$x^3 + y^3 + z^3 - 3xyz = 0$$

at the point $(1, 0, -1)$ is given by

a) $\langle 1, 1, 1 \rangle$

b) $\langle 1, 0, -1 \rangle$

c) $\langle 3, 1, 3 \rangle$

d) $\langle -1, 1, -1 \rangle$

e) $\langle 3, 3, -1 \rangle$

18. Using the linearization $L(x, y)$ of $f(x, y) = x^2y - y^2 - 2y - x^2$ at the point $(1, 2)$, $f(0.9, 2.1)$ is best approximated by

a) -7.7

b) -10

c) -3.7

d) 6

e) 4

19. Let E be the solid region bounded by the sphere $x^2 + y^2 + z^2 = 9$. Evaluate $\displaystyle\iiint_E \frac{z^2}{x^2 + y^2 + z^2}\, dV$

a) $12\,\pi$

b) $121\,\pi$

c) $9\,\pi$

d) $6\,\pi$

e) $\dfrac{9}{2}\,\pi$

20. Let $f(x,y) = x^2 - 2xy + 3y + y^2$ where $x = st^2$ and $y = e^{s-t}$.
$\dfrac{\partial f}{\partial s} + \dfrac{\partial f}{\partial t}$ at $s = 2$ and $t = 1$ is equal to

a) $20 - 10e$
b) $2e^2 - 7e$
c) $3e^2 + 20$
d) $2 - 10e^2$
e) 0

21. If $L = \lim\limits_{(x,y)\to(0,0)} \sqrt{(x^2+y^2)}\,\ln(x^2+y^2)$ then

a) $L = 0$
b) L does not exist
c) $L = 2$
d) $L = 4$
e) $L = 1$