1. If $f(x) = x^2 e^x$, then $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$ $f'(x)$

$= x^2 e^x + e^x \cdot 2x$

$= e^x (x^2 + 2x)$

a) $e^x (x^2 + 2x)$

b) $2x e^x$

c) $2xh e^x$

d) $2x^2 - x$

e) $e^x (x+2)$

2. If $y = \frac{1}{2-\sqrt{x}}$, then $\frac{dy}{dx} =$ $\frac{-\frac{-1}{2\sqrt{x}}}{(2-\sqrt{x})^2} = \frac{1}{2\sqrt{x}(2-\sqrt{x})^2}$

a) $\frac{1}{2\sqrt{x}(2-\sqrt{x})^2}$

b) $\frac{-1}{(2-\sqrt{x})^2}$

c) $\frac{-2}{\sqrt{x}(2-\sqrt{x})}$

d) $\frac{-1}{\sqrt{x}(2-\sqrt{x})^2}$

e) $\frac{1}{2(2-\sqrt{x})^2}$

3. The equation of the tangent line to the curve $y = 2\tan\left(\frac{\pi x}{4}\right)$ at $x = 1$ is

$\cdot\ x = 1 \Rightarrow y = 2\tan(\frac{\pi}{4}) = 2 \cdot 1 = 2 \Rightarrow (1, 2)$

$\cdot\ y' = 2\sec^2(\frac{\pi x}{4}) \cdot \frac{\pi}{4}$

Slope $= y'|_{x=1} = \frac{\pi}{2}\sec^2(\frac{\pi}{4}) = \pi$

Eq of Tangent line is

$y - 2 = \pi(x-1)$

$\Rightarrow y = \pi x + 2 - \pi$

a) $y = \pi x + 2 - \pi$
b) $y = 3\pi x + 2 - 3\pi$
c) $y = x + \dfrac{\pi}{4}$
d) $y = \dfrac{\pi}{4}x + 2 - \dfrac{\pi}{4}$
e) $y = -\pi x + 2 + \pi$

4. The **number** of points at which the curve $y = x^3 - 3x^2 + 4$ has tangent lines parallel to the line $3x + y = 2$ is

$\cdot\ 3x + y = 2 \Rightarrow y = -3x + 2 \Rightarrow$ Slope $= -3$

$\cdot\ y' = 3x^2 - 6x$

So $3x^2 - 6x = -3$

$\Rightarrow x^2 - 2x = -1$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

a) One
b) Two
c) Three
d) Four
e) Zero

5. If $f(x) = \left(\frac{x}{5} - \frac{5}{x}\right)^5$, then $f'(x) =$ $5\left(\frac{x}{5} - \frac{5}{x}\right)^4 \cdot \left(\frac{1}{5} + \frac{5}{x^2}\right)$

$= \left(\frac{x}{5} - \frac{5}{x}\right)^4 \left(1 + \frac{25}{x^2}\right)$

a) $\left(\frac{x}{5} - \frac{5}{x}\right)^4 \left(1 + \frac{25}{x^2}\right)$

b) $5\left(\frac{x}{5} - \frac{5}{x}\right)^4$

c) $\frac{5}{x^2}\left(\frac{x}{5} - \frac{5}{x}\right)^4$

d) $\left(\frac{x}{5} - \frac{5}{x}\right)^4 \left(5 - \frac{1}{x^2}\right)$

e) $\left(\frac{x}{5} - \frac{5}{x}\right)^4 (1 + 5x)$

6. The linearization of $f(x) = e^{\tan^{-1}(3x)}$ at $x = 0$ is given by

a) $L(x) = 1 + 3x$

b) $L(x) = 3x$

c) $L(x) = 3 - x$

d) $L(x) = 2 + x$

e) $L(x) = 1 - 2x$

- $f(0) = e^{\tan^{-1}(0)} = e^0 = 1$
- $f'(x) = e^{\tan^{-1}(3x)} \cdot \frac{1}{1+9x^2} \cdot 3$

$f'(0) = e^0 \cdot \frac{1}{1+0} \cdot 3 = 3$

$L(x) = f(0) + f'(0)(x - 0)$

$= 1 + 3x$

7. The position function of a body moving in a straight line is

$$s(t) = t^3 - 6t^2 + 9t,\ t \geq 0$$

The body **changes direction** at

$v(t) = s'(t) = 0$

$\Rightarrow 3t^2 - 12t + 9 = 0$

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow (t-1)(t-3) = 0$

$\Rightarrow t = 1$ & $t = 3$

a) $t = 1$ and $t = 3$
b) $t = 2$ and $t = 5$
c) $t = 3$ only
d) $t = 1$ and $t = 4$
e) the body never changes direction

$v(t)$: + on $(0,1)$, − on $(1,3)$, + after 3

change of direction at $t=1$ & $t=3$

8. If $y = \log_2 (8\, t^{\ln 2})$, then $\dfrac{dy}{dt} =$

$y = \log_2 8 + \log_2 (t^{\ln 2})$

$= \log_2 8 + (\ln 2) \cdot \log_2 t$

$\dfrac{dy}{dx} = 0 + (\ln 2) \cdot \dfrac{1}{(\ln 2)t} = \dfrac{1}{t}$

a) $\dfrac{1}{t}$
b) t
c) $3 \ln t$
d) $\dfrac{1}{\ln t}$
e) $\log_2 t$

9. If $z = \sqrt[3]{u(u+1)}$ and $u = \dfrac{x}{x-1}$, then $\dfrac{dz}{dx}|_{x=2} =$

$\bullet\ x = 2 \Rightarrow u = \frac{2}{2-1} = 2$

a) $-\dfrac{5}{3\sqrt[3]{36}}$

b) $\dfrac{1}{3\sqrt[3]{36}}$

c) $4\sqrt[3]{36}$

d) $-\dfrac{4}{3\sqrt[3]{36}}$

e) $\dfrac{2}{\sqrt[3]{36}}$

$\bullet\ \frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx}$

$= \frac{1}{3}(u^2+u)^{-2/3} \cdot (2u+1) \cdot \frac{\overbrace{(x-1) - x}^{=-1}}{(x-1)^2}$

$\bullet\ \frac{dz}{dx}\Big|_{\substack{x=2\\u=2}} = \frac{1}{3}(6)^{-2/3} \cdot 5 \cdot -1$

$= \frac{-5}{3} \cdot \frac{1}{\sqrt[3]{36}}$

10. The tangent line to the graph of the curve $y = \ln\left(\dfrac{\sqrt{\tan(2x)}}{1+\sec(2x)}\right)$ at $x = \dfrac{\pi}{6}$ is

a) a horizontal line

b) a vertical line

c) with slope $\dfrac{\sqrt{3}}{3}$

d) with slope $\sqrt{3}$

e) with slope $\dfrac{2\sqrt{3}}{3}$

$y = \frac{1}{2}\ln(\tan(2x)) - \ln(1+\sec(2x))$

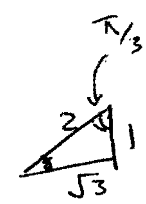
$y' = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\sec^2(2x)}{\tan(2x)} - \frac{2\sec(2x)\tan(2x)}{1+\sec(2x)}$

$y'\Big|_{x=\frac{\pi}{6}} = \frac{4}{\sqrt{3}} - \frac{2 \cdot 2 \cdot \sqrt{3}}{1+2}$

$= \frac{4}{\sqrt{3}} - \frac{4\sqrt{3}}{3}$

$= \frac{4\sqrt{3}}{3} - \frac{4\sqrt{3}}{3}$

$= 0$



11. If $x > 0$, then $\frac{d}{dx}\left[\sin^{-1}\left(\frac{x-4}{x+4}\right)\right] =$ $\frac{1}{\sqrt{1-\left(\frac{x-4}{x+4}\right)^2}} \cdot \frac{(x+4)\cdot 1 - (x-4)\cdot 1}{(x+4)^2}$

$= \frac{x+4}{\sqrt{(x+4)^2-(x-4)^2}} \cdot \frac{8}{(x+4)^2}$

$= \frac{1}{\sqrt{16x}} \cdot \frac{8}{(x+4)}$

$= \frac{1}{4\sqrt{x}} \cdot \frac{8}{(x+4)} = \frac{2}{\sqrt{x}\,(x+4)} = \frac{2\sqrt{x}}{x\,(x+4)}$

a) $\frac{2\sqrt{x}}{x(x+4)}$
b) $\frac{8\sqrt{x}}{x+4}$
c) $\frac{4\sqrt{x}}{x}$
d) $\frac{8}{1+\sqrt{x}}$
e) $(x+4)\sqrt{x}$

12. The radius of a sphere was measured to be $20\,cm$ with a possible error in measurement of at most $0.05\,cm$. The maximum error in the computed volume of the sphere is approximately equal to

$V = \frac{4}{3}\pi r^3$, $r = 20$, $dr = 0.05$

$\Delta V \approx dV = 4\pi r^2\, dr$

$\Delta V \approx 4\pi\,(20)^2\,\frac{5}{100} = 80\pi$

a) $80\,\pi$
b) $60\,\pi$
c) $40\,\pi$
d) $20\,\pi$
e) $10\,\pi$

13. If $5x^5 - y^5 = 1$, then $y'' =$

a) $-\dfrac{20x^3}{y^9}$

b) $\dfrac{5x^3}{y^9}$

c) $-\dfrac{5x^4}{y^4}$

d) $\dfrac{20x^4}{y^8}$

e) $\dfrac{y^5 - 5x^8}{y^9}$

$\bullet\ 25x^4 - 5y^4 \cdot y' = 0$

$\Rightarrow y' = 5\,\dfrac{x^4}{y^4}$

$\bullet\ y'' = 5 \cdot \dfrac{y^4 \cdot 4x^3 - x^4 \cdot 4y^3 \cdot \frac{5x^4}{y^4}}{y^8}$

$= 5 \cdot \dfrac{4x^3y^4 - 20\frac{x^8}{y}}{y^8}$

$= 5 \cdot \dfrac{4x^3y^5 - 20x^8}{y^9}$

$= 5 \cdot 4x^3 \cdot \dfrac{(y^5 - 5x^5)}{y^9} = -\dfrac{20x^3}{y^9}$

$=-1$

14. If $y = \dfrac{\cos x}{e^x}$ then $y'' + 2y' + 3y =$

a) $\dfrac{\cos x}{e^x}$

b) $\dfrac{-\sin x}{e^x}$

c) $\dfrac{\sin x + \cos x}{e^x}$

d) $\dfrac{\sin x}{e^{2x}}$

e) $\dfrac{\sin x - \cos x}{e^{2x}}$

$y = e^{-x}\cos x$

$\bullet\ y' = -e^{-x}\sin x - e^{-x}\cos x$

$\bullet\ y'' = -(e^{-x}\cancel{\cos x} - e^{-x}\sin x) - (-e^{-x}\sin x - \cancel{e^{-x}\cos x})$

$= 2e^{-x}\sin x$

Now

$y'' + 2y' + 3y = 2e^{-x}\sin x - 2e^{-x}\sin x - 2e^{-x}\cos x$

$+ 3e^{-x}\cos x$

$= e^{-x}\cos x = \dfrac{\cos x}{e^x}$

15. Let $f(x) = 1 + 2x - x^2$, $x \leq 1$. Then $\frac{df^{-1}}{dx}|_{x=-2} = \dfrac{1}{\frac{df}{dx}\Big|_{x=f^{-1}(-2)}}$

a) $\frac{1}{4}$

b) $\frac{1}{6}$

c) $-\frac{1}{4}$

d) -1

e) $\frac{1}{3}$

- $f^{-1}(-2) = a \Rightarrow -2 = f(a) = 1 + 2a - a^2$

$\Rightarrow a^2 - 2a - 3 = 0 \Rightarrow (a-3)(a+1) = 0$

$\Rightarrow a = 3$ or $\boxed{a = -1}$

X ✓

- $f'(x) = 2 - 2x \Rightarrow f'(-1) = 2 - 2(-1) = 4$

So the answer is $\frac{1}{4}$.

16. If $y = (\sin x)^{\sqrt[3]{x}}$, then $\frac{y'}{y} =$

a) $\dfrac{3x \cot x + \ln(\sin x)}{3\sqrt[3]{x^2}}$

b) $\sqrt[3]{x}(\sin x)^{\sqrt[3]{x}-1} \cdot \cos x$

c) $\dfrac{x \cot x + 3\ln(\sin x)}{3\sqrt[3]{x}}$

d) $\sqrt[3]{x}\tan x + \ln(\sin x)$

e) $\dfrac{x \tan x - \ln(\sin x)}{\sqrt[3]{x^2}}$

$\ln y = \sqrt[3]{x} \ln(\sin x)$

$\frac{y'}{y} = \sqrt[3]{x} \cdot \frac{\cos x}{\sin x} + \ln(\sin x) \frac{1}{3} x^{-2/3}$

$= \sqrt[3]{x} \cot x + \frac{1}{3} \frac{\ln(\sin x)}{\sqrt[3]{x^2}}$

$= \dfrac{3x \cot x + \ln(\sin x)}{3\sqrt[3]{x^2}}$

17. If the normal line to the curve $x^2 - xy + y^2 = 1$ at $(1, 1)$ **intersects** the curve at another point (a, b), then $a + b =$

• Normal line: $2x - xy' - y + 2yy' = 0$

$\Rightarrow 2 - y' - 1 + 2y' = 0 \Rightarrow y' = -1$

a) -2

b) 2

c) 0

d) 3

e) -1

Slope of the normal $= 1$

Eq. of the normal line is $y - 1 = x - 1 \Rightarrow y = x$

• Pts of intersection : $x^2 - x^2 + x^2 = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\Rightarrow (x, y) = (1, 1), \underline{(-1, -1)}$

Sum $= -1 - 1 = -2$

18. A street light is mounted at the top of a 5-meter-tall pole. A man $2m$ tall walks away from the pole with a speed of $\frac{3}{2}\, m/s$ along a straight path. How fast is **his shadow** moving when he is $10\, m$ from the pole?

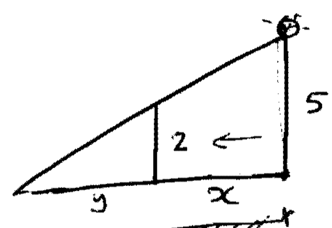
a) $1\, m/s$

b) $2\, m/s$

c) $3\, m/s$

d) $4\, m/s$

e) $5\, m/s$



$\frac{dx}{dt} = \frac{3}{2}$, $\frac{dy}{dt} = ?$ when $x = 10$

$$\frac{y}{2} = \frac{x+y}{5}$$

$\Rightarrow 5y = 2x + 2y$

$\Rightarrow 3y = 2x$

$\Rightarrow y = \frac{2}{3}x$

So $\frac{dy}{dt} = \frac{2}{3}\frac{dx}{dt}$

$= \frac{2}{3} \cdot \frac{3}{2} = 1$

19. If the line $y = 2x + 8$ is a tangent line to the curve $y = \dfrac{c}{x+2}$, then $c^3 - 3c + 4 =$

Let (α, β) be the point of tangency. Then

a) 2
b) 1
c) 0
d) -3
e) 5

$$\left\{\begin{array}{l} \left.\begin{array}{l}\beta = 2\alpha + 8 \\ \beta = \dfrac{c}{\alpha+2}\end{array}\right\} \Rightarrow c = \beta(\alpha+2) = (2\alpha+8)(\alpha+2) \\ 2 = \dfrac{-c}{(\alpha+2)^2} \qquad c = -2(\alpha+2)^2 \end{array}\right\} \Rightarrow \alpha = -3, \ \underset{\text{rejected}}{-2}$$

So $\alpha = -3 \Rightarrow c = -2(-3+1)^2 = -2$

$\Rightarrow c^3 - 3c + 4 = -8 + 6 + 4 = 2$

20. If the function

$$f(x) = \begin{cases} ax + 3 & \text{if } x \geq -1 \\ bx^2 - ax & \text{if } x < -1 \end{cases}$$

is differentiable on $(-\infty, \infty)$, then $f(-2) =$

a) -6
b) -3
c) 0
d) 4
e) 2

- f has to be continuous at $x = -1$:

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$b + a = -a + 3 \Rightarrow \boxed{b + 2a = 3}$ ①

- $f'(x) = \begin{cases} a & \text{if } x > -1 \\ 2bx - a & \text{if } x < -1 \end{cases}$

f has to be diff. at $x = -1$:

right derivat. at $x=-1$ = left deriv. at $x=-1$

$a = -2b - a$

$\Rightarrow \boxed{b = -a}$ ②

Solving ① & ②, we get

$\boxed{a = 3, \ b = -3}$

So $f(-2) = b(-2)^2 - a(-2)$
$= -3(4) - (3)(-2)$
$= -12 + 6$
$= -6$