Solution of Homework 6
Term 123

Chapter 9

9-5 a) α = P(reject H_0 when H_0 is true)

$$= P(\bar{X} \le 11.5 \text{ when } \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{11.5-12}{0.5/\sqrt{4}}\right) = P(Z \le -2)$$

$$= 0.02275.$$

The probability of rejecting the null hypothesis when it is true is 0.02275.

b) β = P(accept H_0 when μ = 11.25) = $P\left(\bar{X} > 11.5 | \mu = 11.25\right)$

$$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{4}}\right) = P(Z > 1.0)$$

$$= 1 - P(Z \le 1.0) = 1 - 0.84134 = 0.15866$$

The probability of accepting the null hypothesis when it is false is 0.15866.

c) β = P(accept H_0 when μ = 11.25) =

$$= P\left(\bar{X} > 11.5 | \mu = 11.5\right) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.5}{0.5/\sqrt{4}}\right)$$

$$= P(Z > 0) = 1 - P(Z \le 0) = 1 - 0.5 = 0.5$$

The probability of accepting the null hypothesis when it is false is 0.5

9-40 a) 1) The parameter of interest is the true mean water temperature, μ.

2) $H_0 : \mu = 38$

3) $H_1 : \mu > 38$

4) $z_0 = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

5) Reject H_0 if $z_0 > z_\alpha$ where $\alpha = 0.05$ and $z_{0.05} = 1.65$

6) $\bar{x} = 37$, $\sigma = 1.1$

$$z_0 = \frac{37-38}{1.1/\sqrt{9}} = -2.73$$

7) Because $-2.73 < 1.65$ fail to reject H_0. The water temperature is not significantly greater than 38 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(-2.73) = 1 - 0.003167 = 0.9968$

c) $\beta = \Phi\left(z_{0.05} + \dfrac{38-40}{1.1/\sqrt{9}}\right)$

$= \Phi(1.65 + -5.45)$

$= \Phi(-3.80) \cong 0.000072$

9-66 In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean impact strength, μ.

2) $H_0 : \mu = 1.0$

3) $H_1 : \mu > 1.0$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,19} = 1.729$ for n = 20

6) $\bar{x} = 1.25$ $s = 0.25$ $n = 20$

$$t_0 = \frac{1.25 - 1.0}{0.25/\sqrt{20}} = 4.47$$

7) Because 4.47 > 1.729 reject the null hypothesis. There is sufficient evidence to conclude that the true mean impact strength is greater than 1.0 ft-lb/in at $\alpha = 0.05$. The P-value < 0.0005

9-67 In order to use a t statistic in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean current, μ.

2) $H_0 : \mu = 300$

3) $H_1 : \mu > 300$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,9} = 1.833$ for n = 10

6) $n = 10$ $\bar{x} = 317.2$ $s = 15.7$

$$t_0 = \frac{317.2 - 300}{15.7/\sqrt{10}} = 3.46$$

7) Because 3.46 > 1.833 reject the null hypothesis. There is sufficient evidence to indicate that the true mean current is greater than 300 microamps at $\alpha = 0.05$. The $0.0025 < P\text{-value} < 0.005$

9-87 a)

1) The parameter of interest is the true fraction of rejected parts

2) $H_0 : p = 0.03$

3) $H_1 : p < 0.03$

4) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

5) Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_\alpha = -z_{0.05} = -1.65$

Chapter 10

10-4 a) 1) The parameter of interest is the difference in fill volume $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 473.581$ $\quad \bar{x}_2 = 473.324$

$\sigma_1 = 0.6$ $\quad \sigma_2 = 0.75$

$n_1 = 10$ $\quad n_2 = 10$

$$z_0 = \frac{(473.581 - 473.324)}{\sqrt{\dfrac{(0.6)^2}{10} + \dfrac{(0.75)^2}{10}}} = 0.85$$

7) Conclusion: Because $-1.96 < 0.85 < 1.96$, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the two machine fill volumes differ at $\alpha = 0.05$.

P-value = $2(1 - \Phi(0.85)) = 2(1 - 0.8023) = 0.395$

b) $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\left(473.581 - 473.324\right) - 1.96\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}} \le \mu_1 - \mu_2 \le \left(473.581 - 473.324\right) + 1.96\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}$$

$$-0.3383 \le \mu_1 - \mu_2 \le 0.8523$$

With 95% confidence, we believe the true difference in the mean fill volumes is between –0.3383 and 0.8523. Because 0 is contained in this interval, we can conclude there is no significant difference between the means.

c) $\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$

$$= \Phi\left(1.96 - \frac{1.2}{\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}}\right) - \Phi\left(-1.96 - \frac{1.2}{\sqrt{\frac{(0.6)^2}{10} + \frac{(0.75)^2}{10}}}\right)$$

$= \Phi(1.96 - 3.95) - \Phi(-1.96 - 3.95) = \Phi(-1.99) - \Phi(-5.91)$ = 0.0233 – 0 = 0.0233

Power = 1 – 0.9481 = 0.0519

10-17 a) 1) The parameter of interest is the difference in mean catalyst yield, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 <$ [ ] where $-t_{0.01,25} = -2.485$ for $\alpha = 0.01$

6) $\bar{x}_1 = 86$ $\bar{x}_2 = 89$ $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 3$ $s_2 = 2$ $= \sqrt{\frac{11(3)^2 + 14(2)^2}{25}} = 2.4899$

$n_1 = 12$ $n_2 = 15$

$$t_0 = \frac{(86-89)}{2.4899\sqrt{\frac{1}{12} + \frac{1}{15}}} = -3.11$$

7) Conclusion: Because –3.11 < –2.485, reject the null hypothesis and conclude that the mean yield of catalyst 2 exceeds that of catalyst 1 at α = 0.01.

b) 99% upper confidence interval $\mu_1 - \mu_2$: $t_{0.01,25} = 2.485$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\mu_1 - \mu_2 \le (86 - 89) + 2.485(2.4899)\sqrt{\frac{1}{12} + \frac{1}{15}}$$

$\mu_1 - \mu_2 \le -0.603$ or equivalently $\mu_1 + 0.603 \le \mu_2$

We are 99% confident that the mean yield of catalyst 2 exceeds that of catalyst 1 by at least 0.603 units.

10-20 a) 1) The parameter of interest is the difference in mean impact strength, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,23} = 1.714$ for $\alpha = 0.05$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 23.21$$

$$\nu \cong 23$$

(truncated)

6) $\bar{x}_1 = 395$ $\quad \bar{x}_2 = 435$

$s_1 = 15$ $\quad s_2 = 30$

$n_1 = 10$ $\quad n_2 = 16$

$$t_0 = \frac{(395 - 435)}{\sqrt{\frac{15^2}{10} + \frac{30^2}{16}}} = -4.51$$

7) Conclusion: Because –4.51 < –1.714 reject the null hypothesis and conclude that supplier 2 provides gears with higher mean impact strength at the 0.05 level of significance.

P-value = $P(t < -4.51)$: P-value < 0.0005

b) 1) The parameter of interest is the difference in mean impact strength, $\mu_2 - \mu_1$

2) $H_0: \mu_2 - \mu_1 = 25$

3) $H_1: \mu_2 - \mu_1 > 25$ or $\mu_2 > \mu_1 + 25$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{\alpha,\nu} = 1.714$ for $\alpha = 0.05$ where

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} = 23.21$$

$$\nu \cong 23$$

6) $\bar{x}_1 = 395$ $\bar{x}_2 = 435$ $\Delta_0 = 35$ $s_1 = 15$ $s_2 = 30$ $n_1 = 10$ $n_2 = 16$

$$t_0 = \frac{(435 - 395) - 35}{\sqrt{\frac{15^2}{10} + \frac{30^2}{16}}} = 0.563$$

7) Conclusion: Because 0.563 < 1.714, fail to reject the null hypothesis. There is insufficient evidence to conclude that the mean impact strength from supplier 2 is at least 35 Nm higher than from supplier 1 using $\alpha = 0.05$.

c) Using the information provided in part (a), and $t_{0.025,25} = 2.069$, a 95% confidence interval on the difference $\mu_2 - \mu_1$ is

$$(\bar{x}_2 - \bar{x}_1) - t_{0.025,25}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_2 - \mu_1 \le (\bar{x}_2 - \bar{x}_1) + t_{0.025,25}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$40 - 2.069(8.874) \le \mu_2 - \mu_1 \le 40 + 2.069(8.874)$$

$$21.64 \le \mu_2 - \mu_1 \le 58.36$$

Because zero is not contained in the confidence interval, we conclude that supplier 2 provides gears with a higher mean impact strength than supplier 1 with 95% confidence.

10-40 b) $\bar{d} = 0.667$ $s_d = 2.964$, $n = 12$

95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$0.667 - 2.201\left(\frac{2.964}{\sqrt{12}}\right) \le \mu_d \le 0.667 + 2.201\left(\frac{2.964}{\sqrt{12}}\right)$$

$$-1.216 \le \mu_d \le 2.55$$

Because zero is contained within this interval, there is no significant indication that one design language is preferable at a 5% significance level

10-68 a) 1) The parameters of interest are the proportion of successes of surgical repairs for different tears, p_1 and p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 > p_2$

4) Test statistic is $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

5) Reject the null hypothesis if $z_0 > z_{0.05}$ where $z_{0.05} = 1.65$ for $\alpha = 0.05$

6) $n_1 = 18$ $n_2 = 30$

$x_1 = 14$ $x_2 = 22$

$\hat{p}_1 = 0.78$ $\hat{p}_2 = 0.73$ $\hat{p} = \dfrac{14+22}{18+30} = 0.75$

$$z_0 = \frac{0.78 - 0.73}{\sqrt{0.75(1-0.75)\left(\frac{1}{18} + \frac{1}{30}\right)}} = 0.387$$

7) Conclusion: Because 0.387 < 1.65 we fail to reject the null hypothesis at the 0.05 level of significance.

P-value = $[1 - P(Z < 0.387)] = 1 - 0.6517 \approx 0.35$