Solution of Homework 5
Term 123

8-4 a) 95% CI for μ, $n = 10,\ \sigma = 25\ \ \bar{x} = 1000,\ z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$
$$1000 - 1.96(25/\sqrt{10}) \le \mu \le 1000 + 1.96(25/\sqrt{10})$$
$$984.5 \le \mu \le 1015.5$$

b) .95% CI for μ, $n = 25,\ \sigma = 25\ \ \bar{x} = 1000,\ z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$
$$1000 - 1.96(25/\sqrt{25}) \le \mu \le 1000 + 1.96(25/\sqrt{25})$$
$$990.2 \le \mu \le 1009.8$$

c) 99% CI for μ, $n = 10,\ \sigma = 25\ \ \bar{x} = 1000,\ z = 2.58$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$
$$1000 - 2.58(25/\sqrt{10}) \le \mu \le 1000 + 2.58(25/\sqrt{10})$$
$$979.6 \le \mu \le 1020.4$$

d) 99% CI for μ, $n = 25,\ \sigma = 25\ \ \bar{x} = 1000,\ z = 2.58$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$
$$1000 - 2.58(25/\sqrt{25}) \le \mu \le 1000 + 2.58(25/\sqrt{25})$$
$$987.1 \le \mu \le 1012.9$$

e) When n is larger, the CI is narrower. The higher the confidence level, the wider the CI.

8-7 a) Find n for the length of the 95% CI to be 40. $Z_{a/2} = 1.96$

$$\text{1/2 length} = (1.96)(20)/\sqrt{n} = 20$$
$$39.2 = 20\sqrt{n}$$
$$n = \left(\frac{39.2}{20}\right)^2 = 3.84$$

Therefore, $n = 4$.

b) Find n for the length of the 99% CI to be 40. $Z_{a/2} = 2.58$

$$\text{1/2 length} = (2.58)(20)/\sqrt{n} = 20$$
$$51.6 = 20\sqrt{n}$$
$$n = \left(\frac{51.6}{20}\right)^2 = 6.66$$

Therefore, $n = 7$.

8-12 99% two-sided CI on the diameter cable harness holes: where $\bar{x} = 3.75$, $\sigma = 0.025$, n = 10 and

$z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{0.005}\sigma/\sqrt{n}$$
$$3.75 - 2.58(0.025)/\sqrt{10} \le \mu \le 3.75 + 2.58(0.025)/\sqrt{10}$$
$$3.73 \le \mu \le 3.77$$

8-27 95% confidence interval on mean tire life

$n = 16 \quad \bar{x} = 60{,}139.7 \quad s = 3645.94 \qquad t_{0.025,15} = 2.131$

$$\bar{x} - t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right)$$
$$60139.7 - 2.131\left(\frac{3645.94}{\sqrt{16}}\right) \le \mu \le 60139.7 + 2.131\left(\frac{3645.94}{\sqrt{16}}\right)$$
$$58197.33 \le \mu \le 62082.07$$

8-35 99% confidence interval on mean current required

Assume that the data are a random sample from a normal distribution.

$n = 10 \quad \bar{x} = 317.2 \quad s = 15.7 \qquad t_{0.005,9} = 3.250$

$$\bar{x} - t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right)$$
$$317.2 - 3.250\left(\frac{15.7}{\sqrt{10}}\right) \le \mu \le 317.2 + 3.250\left(\frac{15.7}{\sqrt{10}}\right)$$
$$301.06 \le \mu \le 333.34$$

8-58 a) 95% confidence interval on the true proportion of helmets showing damage

$$\hat{p} = \frac{18}{50} = 0.36 \qquad n = 50 \qquad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.36 - 1.96\sqrt{\frac{0.36(0.64)}{50}} \leq p \leq 0.36 + 1.96\sqrt{\frac{0.36(0.64)}{50}}$$

$$0.227 \leq p \leq 0.493$$

b) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$

$n \cong 2213$

c) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$