Solution of Homework 3
Term 123

4-4. a) $P(X < 2) = \int_1^2 \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)\Big|_1^2 = \left(\frac{-1}{4}\right) - (-1) = 0.75$

b) $P(X > 5) = \int_5^\infty \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)\Big|_5^\infty = 0 - \left(\frac{-1}{25}\right) = 0.04$

c) $P(4 < X < 8) = \int_4^8 \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)\Big|_4^8 = \left(\frac{-1}{64}\right) - \left(\frac{-1}{16}\right) = 0.0469$

d) $P(X < 4 \, or \, X > 8) = 1 - P(4 < X < 8)$. From part (c), $P(4 < X < 8) = 0.0469$.

Therefore, $P(X < 4 \, or \, X > 8) = 1 – 0.0469 = 0.9531$

e) $P(X < x) = \int_1^x \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)\Big|_1^x = \left(\frac{-1}{x^2}\right) - (-1) = 0.95$

Then, $x^2 = 20$, and x = 4.4721

4-14. a) $P(X < 1.7) = P(X \le 1.7) = F_X(1.7)$ because X is a continuous random variable.

Then, $F_X(1.7) = 0.2(1.7) + 0.5 = 0.84$

b) $P(X > -1.5) = 1 - P(X \le -1.5) = 1 - 0.2 = 0.8$

c) P(X < -2) = 0.1

d) $P(-1 < X < 1) = P(-1 < X \le 1) = F_X(1) - F_X(-1) = 0.7 - 0.3 = 0.4$

4-18. Now, $f(x) = \frac{2}{x^3}$ for x > 1 and

$$F_X(x) = \int_1^x \frac{2}{u^3} du = \left(\frac{-1}{u^2}\right)\Big|_1^x = \left(\frac{-1}{x^2}\right) + 1$$

Then, $F_X(x) = \begin{cases} 0, & x \le 1 \\ 1 - \frac{1}{x^2}, & x > 1 \end{cases}$

4-35. a) $E(X) = \int_{100}^{120} x \frac{600}{x^2} dx = 600 \ln x \Big|_{100}^{120} = 109.39$

$$V(X) = \int_{100}^{120} (x - 109.39)^2 \frac{600}{x^2} dx = 600 \int_{100}^{120} 1 - \tfrac{2(109.39)}{x} + \tfrac{(109.39)^2}{x^2} dx$$

$$= 600(x - 218.78 \ln x - 109.39^2 x^{-1}) \Big|_{100}^{120} = 33.19$$

b.) Average cost per part = $0.50*109.39 = $54.70

4-41. a) The distribution of X is $f(x)$ = 6.67 for 0.90 < x < 1.05. Now,

$$F_X(x) = \begin{cases} 0, & x < 0.90 \\ 6.67x - 6, & 0.90 \le x < 1.05 \\ 1, & 1.05 \le x \end{cases}$$

b) $P(X > 1.02) = 1 - P(X \le 1.02) = 1 - F_X(1.02) = 0.2$

c) If P(X > x) = 0.90, then 1 – F(X) = 0.90 and F(X) = 0.10. Therefore, 6.67x – 6 = 0.10 and x = 0.915.

d) E(X) = (1.05 + 0.9)/2 = 0.975 and V(X) = $\frac{(1.05 - 0.9)^2}{12} = 0.00188$

4-49. a) P(Z<1.32) = 0.90658

b) P(Z<2.0) = 0.97725

c) P(Z>1.45) = 1 – 0.92647 = 0.07353

d) P(Z > –2.15) = p(Z < 2.15) = 0.98422

e) P(–2.34 < Z < 1.76) = P(Z<1.76) – P(Z > 2.34) = 0.95116

4-51. a) P(Z < 1.28) = 0.90

b) P(Z < 0) = 0.5

c) If P(Z > z) = 0.1, then P(Z < z) = 0.90 and z = 1.28

d) If P(Z > z) = 0.9, then P(Z < z) = 0.10 and z = –1.28

e) P(–1.24 < Z < z) = P(Z < z) – P(Z < –1.24)

= P(Z < z) – 0.10749.

Therefore, P(Z < z) = 0.8 + 0.10749 = 0.90749 and z = 1.33

4-55. a) P(X < 11) = $P\left(Z < \frac{11-5}{4}\right)$

= P(Z < 1.5)

= 0.93319

b) P(X > 0) = $P\left(Z > \frac{0-5}{4}\right)$

= P(Z > –1.25)

= 1 – P(Z < –1.25)

= 0.89435

c) P(3 < X < 7) = $P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$

= P(–0.5 < Z < 0.5)

= P(Z < 0.5) – P(Z < –0.5)

= 0.38292

d) P(–2 < X < 9) = $P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$

= P(–1.75 < Z < 1)

= P(Z < 1) – P(Z < –1.75)]

= 0.80128

e) P(2 < X < 8) = $P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$

= P(–0.75 < Z < 0.75)

= P(Z < 0.75) – P(Z < –0.75)

= 0.54674

4-58. (a) Let X denote the time.

$X \sim N(260, 50^2)$

$$P(X > 240) = 1 - P(X \le 240) = 1 - \Phi\left(\frac{240-260}{50}\right) = 1 - \Phi(-0.4) = 1 - \ 0.3446 = 0.6554$$

(b) $\Phi^{-1}(0.25) \times 50 + 260 = 226.2755$

$\Phi^{-1}(0.75) \times 50 + 260 = 293.7245$

(c) $\Phi^{-1}(0.05) \times 50 + 260 = 177.7550$

4-68. Let X denote the demand for water daily.

X ~ N(1170, 170^2)

(a) $P(X > 1320) = 1 - P(X \le 1320) = 1 - \Phi\left(\frac{1320-1170}{170}\right) = 1 - \Phi\left(\frac{150}{170}\right) = 0.1894$

(b) $\Phi^{-1}(0.99) \times 170 + 1170 = 1566.1$

(c) $\Phi^{-1}(0.05) \times 170 + 1170 = 891.2$

(d) $X \sim N(\mu,\ 170^2)$

$$P(X > 1320) = 1 - P(X \le 1320) = 1 - \Phi\left(\frac{1320-\mu}{170}\right) = 0.01$$

$$\Phi\left(\frac{1320-\mu}{170}\right) = 0.99$$

$$\mu = 1320 - \Phi^{-1}(0.99) \times 170 = 923.9$$

4-69. a) $P(X < 5000) = P\left(Z < \frac{5000-7000}{600}\right)$

$= P(Z < -3.33) = 0.00043.$

b) P(X > x) = 0.95. Therefore, $P\left(Z > \frac{x-7000}{600}\right) = 0.95$ and $\frac{x-7000}{600} = -1.64.$

Consequently, x = 6016.

c) $P(X > 7000) = P\left(Z > \frac{7000-7000}{600}\right) = P(Z > 0) = 0.5$

P(three lasers operating after 7000 hours) = $(1/2)^3 = 1/8$

4-70. a) P(X > 0.0065) = $P\left(Z > \frac{0.0065 - 0.005}{0.001}\right)$

= P(Z > 1.5)

= 1-P(Z < 1.5)

= 0.06681.

b) P(0.0035 < X < 0.0065) = $P\left(\frac{0.0035 - 0.005}{0.001} < Z < \frac{0.0065 - 0.005}{0.001}\right)$

= P(−1.5 < Z < 1.5)

= 0.86638.

c) P(0.0035 < X < 0.0065) = $P\left(\frac{0.0035 - 0.005}{\sigma} < Z < \frac{0.0065 - 0.005}{\sigma}\right)$

= $P\left(\frac{-0.0015}{\sigma} < Z < \frac{0.0015}{\sigma}\right)$.

Therefore, $P\left(Z < \frac{0.0015}{\sigma}\right)$ = 0.9975. Therefore, $\frac{0.0015}{\sigma}$ = 2.81 and σ = 0.000534.

4-80. Let X denote the number of defective chips in the lot.

Then, E(X) = 1000(0.02) = 20, V(X) = 1000(0.02)(0.98) = 19.6.

a) $P(X > 25) \cong P\left(Z > \frac{25.5 - 20}{\sqrt{19.6}}\right) = P(Z > 1.24) = 1 - P(Z \le 1.24) = 0.107$

b)

$$P(20 < X < 30) \cong P(20.5 < X < 29.5) = P\left(\frac{.5}{\sqrt{19.6}} < Z < \frac{9.5}{\sqrt{19.6}}\right) = P(0.11 < Z < 2.15)$$

$$= 0.9842 - 0.5438 = 0.44$$

4-86. X is the number of minor errors on a test pattern of 1000 pages of text. X is a Poisson random variable with a mean of 0.4 per page

a) The numbers of errors per page are random variables. The assumption that the occurrence of an event in a subinterval in a Poisson process is independent of events in other subintervals implies that the numbers of events in disjoint intervals are independent. The pages are disjoint intervals and consequently the error counts per page are independent.

b) $P(X = 0) = \dfrac{e^{-0.4}0.4^0}{0!} = 0.670$

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.670 = 0.330$

The mean number of pages with one or more errors is 1000(0.330) = 330 pages

c) Let Y be the number of pages with errors.

$$P(Y > 350) \cong P\left(Z \geq \frac{350.5 - 330}{\sqrt{1000(0.330)(0.670)}} \right) = P(Z \geq 1.38) = 1 - P(Z < 1.38)$$
$$= 1 - 0.9162 = 0.0838$$

4-99. Let X denote the time until the arrival of a taxi. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.1$ arrivals/ minute.

a) P(X > 60) = $\int_{60}^{\infty} 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_{60}^{\infty} = e^{-6}$ = 0.0025

b) P(X < 10) = $\int_{0}^{10} 0.1e^{-0.1x}dx = -e^{-0.1x}\Big|_{0}^{10} = 1 - e^{-1}$ = 0.6321

c) P(X > x) = $\int_{x}^{\infty} 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_{x}^{\infty} = e^{-0.1x} = 0.15$ and x = 18.97 minutes.

d) P(X < x) = 0.9 implies that P(X > x) = 0.1. Therefore, this answer is the same as part c).

e) P(X < x) = $-e^{-0.1t}\Big|_{0}^{x} = 1 - e^{-0.1x} = 0.5$ and x = 6.93 minutes.

4-105. Let X denote the number of calls in 30 minutes. Because the time between calls is an exponential random variable, X is a Poisson random variable with $\lambda = 1/E(X) = 0.1$ calls per minute = 3 calls per 30 minutes.

a) P(X > 3) = $1 - P(X \le 3) = 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!}\right] = 0.3528$

b) P(X = 0) = $\frac{e^{-3}3^0}{0!} = 0.04979$

c) Let Y denote the time between calls in minutes. Then, $P(Y \ge x) = 0.02$ and

$$P(Y \ge x) = \int_x^\infty 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_x^\infty = e^{-0.1x}.$$ Therefore, $e^{-0.1x} = 0.02$ and x = 39.12 minutes.

. d) P(Y > 120) = $\int_{120}^\infty 0.1e^{-0.1y}dy = -e^{-0.1y}\Big|_{120}^\infty = e^{-12} = 6.14\times10^{-6}$.

e) Because the calls are a Poisson process, the numbers of calls in disjoint intervals are independent. From Exercise 4-90 part b), the probability of no calls in one-half hour is $e^{-3} = 0.04979$. Therefore, the answer is $\left[e^{-3}\right]^4 = e^{-12} = 6.14\times10^{-6}$. Alternatively, the answer is the probability of no calls in two hours. From part d) of this exercise, this is e^{-12}.

f) Because a Poisson process is memoryless, probabilities do not depend on whether or not intervals are consecutive. Therefore, parts d) and e) have the same answer.