Solution of Homework 2

Term 123

3-4. The range of X is $\{0,1,2,3,4,5\}$

3-5. The range of X is $\{1,2,...,591\}$ . Because 590 parts are conforming, a nonconforming part must be selected in 591 selections.

3-12. The range of X is {100, 101, …, 150}

3-17. Probabilities are nonnegative and sum to one.

a) P(X = 3) = 7/25

b) P(X ≤ 1) = 1/25 + 3/25 = 4/25

c) P(2 ≤ X < 4) = 5/25 + 7/25 = 12/25

d) P(X > –10) = 1

3-21. X = number of wafers that pass

P(X = 0) = $(0.3)^3$ = 0.027

P(X = 1) = $3(0.3)^2(0.7)$ = 0.189

P(X = 2) = $3(0.3)(0.7)^2$ = 0.441

P(X = 3) = $(0.7)^3$ = 0.343

3-35.

$$F(x)=\begin{cases} 0, & x<0 \\ 0.008, & 0\le x<1 \\ 0.104, & 1\le x<2 \\ 0.488, & 2\le x<3 \\ 1, & 3\le x \end{cases}$$

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$$f(0) = 0.2^3 = 0.008,$$
$$f(1) = 3(0.2)(0.2)(0.8) = 0.096,$$
$$f(2) = 3(0.2)(0.8)(0.8) = 0.384,$$
$$f(3) = (0.8)^3 = 0.512,$$

3-42. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;

pmf: f(1/8) = 0.2, f(1/4) = 0.7, f(3/8) = 0.1

a) $P(X \leq 1/18) = 0$

b) $P(X \leq 1/4) = 0.9$

c) $P(X \leq 5/16) = 0.9$

d) $P(X > 1/4) = 0.1$

e) $P(X \leq 1/2) = 1$

3-58. μ=E(X)=350*0.06 + 450*0.1 + 550*0.47 + 650*0.37 = 565

V(X)= $\sum_{i=1}^{4} f(x_i)(x-\mu)^2$ =6875

σ= $\sqrt{V(X)}$ =82.92

3-65. E(X) = (5 + 1)/2 = 3, V(X) = [(5 – 1 + 1)$^2$ – 1 ]/12 = 2

3-87. Let X denote the number of mornings the light is green.

a) $P(X=1)=\binom{5}{1}0.2^1 0.8^4=0.410$

b) $P(X=4)=\binom{20}{4}0.2^4 0.8^{16}=0.218$

c) $P(X>4)=1-P(X\le 4)=1-0.630=0.370$

3-105. Let X denote the number of calls needed to obtain a connection.

Then, X is a geometric random variable with p = 0.03.

a) $P(X=10)=(1-0.03)^9 0.03=0.97^9 0.03=0.0228$

b) $P(X>5)=1-P(X\le 4)=1-[P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)]$

$$=1-[0.03+0.97(0.03)+0.97^2(0.03)+0.97^3(0.03)+0.97^4(0.03)]$$

$$=1-0.1413=0.8587$$

May also use the fact that P(X > 5) is the probability of no connections in 5 trials. That is,

$$P(X>5)=\binom{5}{0}0.03^0 0.97^5=0.8587$$

c) E(X) = 1/0.03 = 33.33

3-122. Let X denote the number of cards in the sample that are defective.

a) $P(X\ge 1)=1-P(X=0)$

$$P(X=0)=\frac{\binom{20}{0}\binom{130}{20}}{\binom{150}{20}}=\frac{\frac{130!}{20!10!}}{\frac{150!}{20!130!}}=0.04609$$

$$P(X\ge 1)=1-0.04609=0.95391$$

b) $P(X\ge 1)=1-P(X=0)$

$$P(X=0)=\frac{\binom{5}{0}\binom{145}{20}}{\binom{150}{20}}=\frac{\frac{145!}{20!125!}}{\frac{150!}{20!130!}}=\frac{145!130!}{125!150!}=0.4838$$

$$P(X\ge 1)=1-0.4838=0.5162$$

3-137. a) $E(X) = \lambda = 0.2$ errors per test area

b) $P(X \le 2) = e^{-0.2} + \frac{e^{-0.2} 0.2}{1!} + \frac{e^{-0.2} (0.2)^2}{2!} = 0.9989$

99.89% of test areas