KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 123

STAT 319 Statistics for Engineers and Scientists

Third Major Exam | Sunday July 21, 2013

Please check/circle your instructor's name

□ Anabosi □ Jabbar □ Al-Sabah □ Saleh □ Alsawi

Name: Key ID #: ______ Section# ______

☺ **Important Note:**

Show all your work

a) including formulas,
b) intermediate steps, and
c) Final answer.
d) In testing problems, write down
   i) The null and alternative hypotheses
   ii) The test statistic
   iii) The rejection region
   iv) The decision, and
   v) The conclusion

Question No	Full Marks	Marks Obtained
1	10	
2	10	
3	2	
4	13	
Total	35	

1) 44 water samples were collected from a lake and analyzed for concentration of both lead and aluminum particles.

a) The lead concentration measurements had a mean of 9.9 nmol/l and a standard deviation of 8.4 nmol/l. Calculate a 95% confidence interval for the true mean lead concentration in water samples collected from the lake. *(3 pts)*

$$1-\alpha = 0.95, \qquad \alpha = 0.05, \qquad z_{\frac{0.05}{2}} = 1.96$$ (1 pt)

The confidence interval is given by $\bar{x} \pm z_{0.025}\frac{s}{\sqrt{n}}$ → (1 pt)

$$9.9 \pm (1.96)\frac{8.4}{\sqrt{44}}$$

$$9.9 \pm 2.48$$

$$7.42\ noml/l \le \mu \le 12.38\ noml/l$$ (1 pt)

b) Do you need any assumptions to construct the interval in a)? If yes, what are they? If no, why? *(1 pt)*

No assumptions are needed since n large and thus $\overline{X}$ is approximately normal. (1 pt)

c) The aluminum concentration measurements had a mean of 6.7 nmol/l and standard deviation of 10.8 nmol/l. Construct a 99% confidence interval for the difference between the true means between lead concentration and the aluminum concentration in water samples collected from the lake. *(3 pts)*

$$-\alpha = 0.99, \qquad \alpha = 0.01, \qquad z_{\frac{0.01}{2}} = 2.575$$ ← (1 pt)

The confidence interval is given by

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\frac{0.01}{2}}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$ (1 pt)

$$(9.9 - 6.7) \pm (2.575)\sqrt{\frac{(8.4)^2}{44} + \frac{(10.8)^2}{44}}$$

$$9.9 \pm 5.31$$

$$-2.11\ nmol/l \le \mu \le 8.51\ nmol/l$$ (1 pt)

d) Based on the result in c) can you conclude that there is a difference between lead and aluminum concentration in water samples. Justify your answer. *(2 pts)*

(1 pt) (1 pt)

Since 0 is in the interval we conclude there is no evidence that aluminum and lead concentration are different at the 1% significance level.

e) At what significance level are you making the conclusion in d)? *(1 pt)*

1% significance level (1 pt)

2) Chewing gum packages are labeled as 6 ounces, but the company wants the packages to contain a mean of 6.17 ounces. A sample of 20 packages is selected periodically, and the packaging process is stopped if there is evidence that the mean amount packaged is less than from 6.17 ounces. Suppose that in a particular sample of 20 packages, the mean amount dispensed is 6.157 ounces, with a standard deviation of 0.042 ounces.

a) At the 10% significance level, is there evidence to stop the process? *(7 pts)*

H_0: $\mu \geq 6.17$ vs. H_A: $\mu < 6.17$ (1 pt)

Test Statistic $t = \frac{(\bar{x}-\mu_0)}{s/\sqrt{n}}$ (1 pt)

Reject H_0 if $t_{19} < -1.328$ (1 pt)

The observed test statistics $t_0 = -1.38$ (1 pt)

Since t_0 lies in the regression region, we reject H_0, (1 pt)

and we conclude that there is evidence that the mean amount packaged is less than from 6.17 ounces (1 pt)

Therefore the process should be stopped (1 pt)

b) Determine the p-value and explain how it can be used? *(3 pts)*

$p - value = P(t_{19} < -1.38)$ (1 pt)

$0.05 <$ *p-value* < 0.1 (1 pt)

Thus for any $\alpha \geq 0.1$, we reject H_0 (1 pt)

3) An estimate of the mean time until a machine requires service is desired. If it can be assumed that the standard deviation is 60 days, how large a sample is needed so that one will be able to say with 90% confidence that the sample mean is off by at most 10 days? *(2 pts)*

$$n \geq \left(\frac{z_{\frac{0.1}{2}}S}{e}\right)^2 = \left(\frac{(1.645)(60)}{10}\right)^2 = 97.535$$ (1 pt)

The minimum size needed is 98 (1 pt)

4) A new radar device is being considered for a certain defense missile system. The system is checked by experimenting with actual aircraft in which a kill or a no kill is simulated. The existing system has a kill probability of 80%

a) If in 300 trials, 250 kills occur, at the 0.04 level of significance, test the claim that the new system is better than the existing one. *(7 pts)*

$$H_0:\ p = 0.8 \quad vs. \quad H_1:\ p > 0.8$$ (1 pt)

Since $np_0 = 240 \quad and\ n(1-p_0) = 60$ (1 pt)

The test statistic

$$Z = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$\hat{p} = \frac{250}{300} = \frac{250}{300} = \frac{5}{6} = 0.8333$$ (1 pt)

Reject H_0 if $Z > z_{0.04} = 1.75$ (1 pt)

The observed test statistic

$$Z_0 = \frac{\left(\frac{5}{6} - 0.8\right)}{\sqrt{\frac{0.8(1-0.8)}{300}}} = 1.44$$ (1 pt)

Since $Z_0 = 1.44 < z_{0.04} = 1.75$, we don't reject H_0 (1 pt)

And we conclude that there is no evidence that that the new system is better than the existing one. (1 pt)

b) What is the p-value of the test in a)? *(2 pt)*

1 pt

p-value $= P(Z > 1.44) = 0.0749$ (1 pt)

c) Construct a 90% C.I. for the probability of a kill with the new system and interpret it. *(4 pts)*

$$\bar{p} \pm z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{5}{6} \pm 1.645\sqrt{\frac{\frac{5}{6}\left(1-\frac{5}{6}\right)}{300}} = \frac{5}{6} \pm (1.645)(0.021)$$

(1 pt)

$$\frac{5}{6} \pm 0.0345$$ (1 pt)

$$0.798 \leq p \leq 0.868$$ (1 pt)

With 90% we confident that the new system will kill between 79.8% and 86.8% (1 pt)