1) Given that the cumulative distribution function of T, the number of years until a machine fails, is

$$F(t) = \begin{cases} 0, & t < 1 \\ 1/4, & 1 \le t < 3 \\ 2/3, & 3 \le t < 5 \\ 3/4, & 5 \le t < 7 \\ 1.0, & 7 \le t \end{cases}$$

a) Find *(3pts)*

i) P(T = 5)

$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$ (1 pt)

ii) P(T > 3)

$= 1 - P(T \le 3) = 1 - \frac{2}{3} = \frac{1}{3}$ (1 pt)

iii) P(1.4 < T < 6).

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ (1 pt)

b) Find the probability mass function; *f(t)*. *(3pts)*

t	1	3	5	7
$f(t)$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
	(1 pt)	(1 pt)		(1 pt)

2) A geologist has collected 5 specimens of basaltic rock and 10 specimens of granite. He randomly selects 3 of the specimens for analysis. What is the probability that all specimens selected come from one type of rock? *(5pts)*

P(All specimens come from basaltic rock)

(1 pt) $= \frac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = \frac{10}{455}$ (1 pt)

P(All specimens come from granite) $= \frac{\binom{5}{0}\binom{10}{3}}{\binom{15}{3}} = \frac{120}{455}$ (1 pt) (1 pt)

$\Rightarrow$ P(All specimens come from one type of rock)

$= \frac{10}{455} + \frac{120}{455} = \frac{130}{455}$ (1 pt)

3) A computer software firm has been told by its local electric company that there is a 25 percent chance that the electricity will be shut off the next working day. The company estimates that it will cost $400 in lost revenues if employees do not use their computers the next day, and it will cost $1200 if the employees suffer a cutoff in power while using them. What is a better strategy for the company, to not use the computers or use them and risk a shut off? Justify your answer. *(4pts)*

Hint: Define a random variable, and use its properties to answer the question.

Let X = cost if computers are used

1 pt $X = \begin{cases} 1200 & \text{with probability } 0.75 \\ 0 & \end{cases}$ (0.25)

1 pt Expected cost if computers are used $= E(X) = (1200)(0.75)$
$= \$300$.

1 pt Let Y = cost if computers are not used
$= \$400$

1 pt Since $E(X) < E(Y)$ then a better strategy is to use the computers & risk a shutoff

4) Each CD produced by a certain company will be defective with probability 0.05 independent of the others. The company sells the CDs in packages of 4, and returns the money to the customer if the package has any defective CD.

a) What is the probability that a package is returned? *(3pts)*

X = # of defective CD's in a package

$P(\text{package is returned}) = P(X \geq 1)$ (1 pt)

$= 1 - P(X = 0)$

$= 1 - \binom{4}{0}(0.05)^0(0.95)^4$ (1 pt)

$= 1 - 0.8145 = 0.1855$ (1 pt)

b) If a customer buys 3 packages, what is the probability that exactly one of them is returned? *(2pts)*

Y = # of returned package

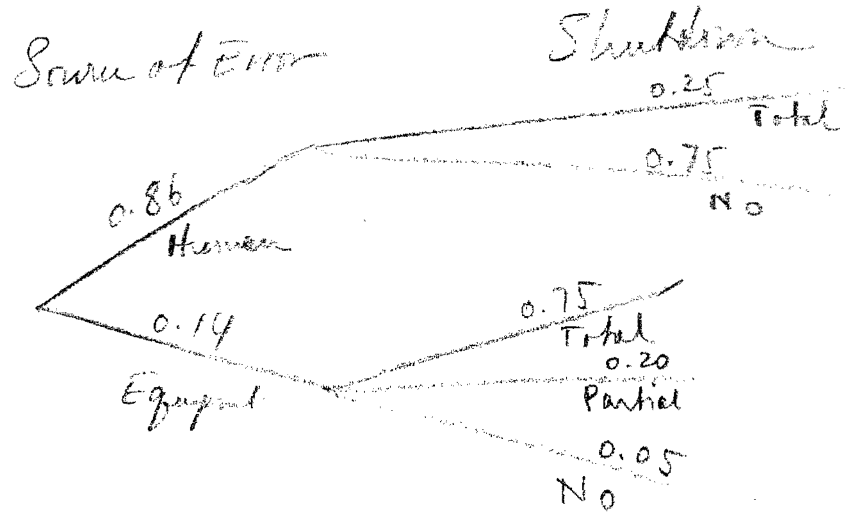
$Y \sim B(3, 0.1855)$ (1 pt)

$P(Y = 1) = \binom{3}{1}(0.1855)(0.8145)^2$

$= 0.3692$ (1 pt)

5) 86 % of process failures are due to human error, and the rest is due to equipment factors. Equipment factors result in a total shutdown 75% of the time, or a partial shutdown 20% of the time or no shutdown. On the other hand, a human error results in a total shutdown 25% of the time, or no shutdown.

a) What is the probability of a total shutdown? *(3pts)*



(1pt)

$P(\text{total shutdown}) = (0.86)(0.25) + (0.14)(0.75)$ (1pt)

$= 0.215 + 0.105 = 0.320$ (1pt)

b) If the process is totally shutdown, what is the probability that the cause was human error? *(2pts)*

$P(\text{Human error} \mid \text{Total Shutdown})$

$= \dfrac{P(\text{Human error and total shutdown})}{P(\text{total shutdown})}$ (1pt)

$= \dfrac{0.215}{0.320}$

$= 0.671$ (1pt)