1) An evaluation of trace metal chemistry in an acidic lake was reported. 44 water samples were collected from Darts Lake, New York, and analyzed for concentration of both lead and aluminum particles.

i) The lead concentration measurements had a mean of 9.9 nmol/l and standard deviation of 8.4 nmol/l. Calculate a 99% confidence interval for the true mean lead concentration in water samples collected from Darts Lake.

$$\mathbf{1-\alpha=0.99,\qquad \alpha=0.01,\qquad z_{\frac{0.01}{2}}=2.575}$$

The confidence interval is given by

$$\bar{x}\pm z_{\frac{0.01}{2}}\frac{s}{\sqrt{n}}$$

$$9.9\pm(2.575)\frac{8.4}{\sqrt{44}}$$

$$9.9\pm 3.260845$$

$$6.639155\ noml/l\ \le\ \mu\ \le 13.16085\ noml/l$$

ii) Do you need any assumptions to construct the interval in a)? If yes, what are they?

Yes,

The population is normal and the standard deviation is unknown and $n=44$

iii) The aluminum concentration measurements had a mean of 6.7 nmol/l and standard deviation of 10.8 nmol/l. Construct a 99% confidence interval for the difference between the true means between lead concentration and the aluminum concentration in water samples collected from Darts Lake.

$$\mathbf{1-\alpha=0.99,\qquad \alpha=0.01,\qquad z_{\frac{0.01}{2}}=2.575}$$

The confidence interval is given by

$$(\bar{x}_1-\bar{x}_2)\pm z_{\frac{0.01}{2}}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

$$(9.9-6.7)\pm(2.5745)\sqrt{\frac{(8.4)^2}{44}+\frac{(10.8)^2}{44}}$$

$$9.9\pm(2.575)(2.062655)$$

$$-2.11134\ nmol/l\ \le\ \mu\ \le 8.511337\ nmol/l$$

iv) Use the result in c) to test whether there is a difference between the lead and aluminum concentration in water samples. At what significance level are you performing this test?

Since $0\in(-2.11134,\ 8.511337\ )$, no there is no difference between the lead and aluminum concentration in water samples

2) A chewing gum manufacturer uses machines to package chewing gum as they move along a filling line. Although the packages are labeled as 6 ounces, the company wants the packages to contain a mean of 6.17 ounces. A sample of 30 packages is selected periodically, and the packages process is stopped if there is evidence that the mean amount packaged is less than from 6.17 ounces. Suppose that in a particular sample of 30 packages, the mean amount dispensed is 6.157 ounces, with a standard deviation of 0.042 ounces.

i) At the 10% significance level, is there evidence to stop the process?

1. **The assumption**

   The population normal
   The standard deviation unknown
   The sample size is 30 large

2. **The hypothesis**

$$H_0:\ \mu \geq 6.17 \quad vs. \quad H_A:\ \mu < 6.17$$

3. **The test statistic**

$$Z_{STAT} = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s} = \frac{(6.157 - 6.17)\sqrt{30}}{0.042} = -1.69533$$

4. **The decision rule and the critical values**

   Reject H_0 if $Z_{STAT} < -z_{0.1} = -1.28$

5. **The decision**

   Since $Z_{STAT} = -1.69533 < -z_{0.1} = -1.28$, reject H_0

6. **The conclusion**

   Yes, there is evidence that the mean amount packaged is less than from 6.17 ounces

ii) Determine the p-value and interpret its meaning.

$$p - value = P(Z < z_{stat}) = P(Z < -1.69533) = 0.0455$$

It is the probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true. Also called observed level of significance.

3) An estimate of the mean time until a machine requires service is desired. If it can be assumed that the standard deviation is 60 days, how large a sample is needed so that one will be able to say with 90% confidence that the sample mean is off by at most 10 days?

$$n \geq \left(\frac{z_{\frac{0.1}{2}} s}{e}\right)^2 = \left(\frac{(1.646)(60)}{10}\right)^2 = 97.535$$

The minimum size needed is 98

4) A new radar device is being considered for a certain defense missile system. The system is checked by experimenting with actual aircraft in which a kill or a no kill is simulated. The existing system has a kill probability of 80%

i) If in 300 trials, 250 kills occur, accept or reject, at the 0.04 level of significance, the claim that the new system is better than the existing one.

1. The assumption

$$np_0 = 240 \quad and\, n(1-p_0) = 60$$

2. The hypothesis

$$H_0\colon\ p \le 0.8 \quad vs. \quad H_A\colon\ p > 0.8$$

3. The test statistic

$$\bar{p} = \frac{250}{300} = \frac{250}{300} = \frac{5}{6} = 0.8333$$

$$Z_{STAT} = \frac{(\bar{p} - p_0)}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\left(\frac{5}{6} - 0.8\right)}{\sqrt{\frac{0.8(1-0.8)}{300}}} = 1.443376$$

4. The decision rule and the critical values

Reject H_0 if $Z_{STAT} > z_{0.04} = 1.75$

5. The decision

Since $Z_{STAT} = 1.443376 < z_{0.04} = 1.75$, don't reject H_0

6. The conclusion

Yes, there is no evidence that that the new system is better than the existing one.

ii) Construct a 99% C.I. for the probability of a kill with the new system and interpret it.

$$1 - \alpha = 0.99, \qquad \alpha = 0.01, \qquad z_{\frac{0.01}{2}} = 2.575$$

The confidence interval is given by

$$\bar{p} \pm z_{\frac{0.01}{2}}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\frac{5}{6} \pm (2.575)\sqrt{\frac{\frac{5}{6}\left(1-\frac{5}{6}\right)}{300}}$$

$$\frac{5}{6} \pm (2.575)(0.021517)$$

$$\frac{5}{6} \pm 0.055405$$

$$0.777928 \le p \le 0.888739$$

With 99% we confident that the new system will kill between 78% and 89%