KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 123

STAT 319 Statistics for Engineers and Scientists

First Major Exam Sunday June 23, 2013

Please check/circle your instructor's name

□ Anabosi □ Jabbar □ Al-Sabah □ Saleh □ Alsawi

Name: _____ ID #: _____Section# _____

☺ Important Note:

Show all your work including formulas, intermediate steps and final answer.

Question No	Full Marks	Marks Obtained
1	6	
2	5	
3	4	
4	5	
5	5	
Total	25	

1) Given that the cumulative distribution function of *T*, the number of years until a machine fails, is

$$F(t) = \begin{cases} 0\,, & t < 1 \\ 1/4, & 1 \le t < 3 \\ 2/3, & 3 \le t < 5 \\ 3/4, & 5 \le t < 7 \\ 1.0, & 7 \le t \end{cases}$$

a) Find *(3pts)*

i) P(T = 5) $= \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

ii) P(T > 3) $= 1 - F(3) = 1 - \frac{2}{3} = \frac{1}{3}$

iii) P(1.4 < T < 6) $= F(6) - F(1.4) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

b) Find the probability mass function; *f(t).* *(3pts)*

T	1	3	5	7
$f(t) = P(T = t)$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{4}$

2) A geologist has collected 5 specimens of basaltic rock and 10 specimens of granite. He randomly selects 3 of the specimens for analysis. What is the probability that all specimens selected come from one type of rock? *(5pts)*

Let X r.v refers to number of basaltic rock, and let Y r.v refer to number of specimens

Since the selection from finite population, the r.v's follows Hypergeometric distribution

X = 0, 1, 2, 3 and Y = 0, 1, 2, 3

$$P(All\ basaltic) + P(All\ granite) = P(X = 3) + P(Y = 3) = \frac{C_3^5 C_0^{10}}{C_3^{15}} + \frac{C_0^5 C_3^{10}}{C_3^{15}}$$

$$= \frac{C_3^5 C_0^{10}}{C_3^{15}} + \frac{C_0^5 C_3^{10}}{C_3^{15}} = \frac{10}{455} + \frac{120}{455} = \frac{130}{455} = \frac{2}{7}$$

3) A computer software firm has been told by its local electric company that there is a 25 percent chance that the electricity will be shut off the next working day. The company estimates that it will cost $400 in lost revenues if employees do not use their computers the next day, and it will cost $1200 if the employees suffer a cutoff in power while using them. What is a better strategy for the company, to not use the computers or use them and risk a shut off? Justify your answer. *(4pts)*

Hint: Define a random variable, and use its properties to answer the question.

Let X random variable refer to cost if computers are used

$$X = \begin{cases} \$1200 & with\ probability\ = 0.25 \\ \$0 & with\ probability\ = 0.75 \end{cases}$$

The expected loss value if the computers used: $E(X) = \$1200(0.25) = \$300$

Let Y random variable refer to cost if computers are not used

$$Y = \$400\ \ with\ probability\ = 1$$

The expected loss value if the computers not used: $E(Y) = \$400$

Since $E(X) < E(Y)$ the firm should risk using their computers

4) Each CD produced by a certain company will be defective with probability 0.05 independent of the others. The company sells the CDs in packages of 4, and returns the money to the customer if the package has any defective CD.

a) What is the probability that a package is returned? *(3pts)*

X r.v refer to number of defective CD's, $X = 0, 1, 2, 3, 4$
X has a Binomial distribution with $p = 0.05$ and $n = 4$
The package is returned if it has at least one defective CD

$$P(X \geq 1) = 1 - P(X = 0)$$
$$= 1 - C_0^4(0.05)^0(1 - 0.05)^4$$
$$= 1 - 0.8145 = 0.1854$$

b) If a customer buys 3 packages, what is the probability that exactly one of them is returned? *(2pts)*

Y r.v refer to number of packages returned, $Y = 0, 1, 2, 3$ and $p = 0.1854$
Y has a Binomial distribution with $p = 0.1854$ and $n = 3$

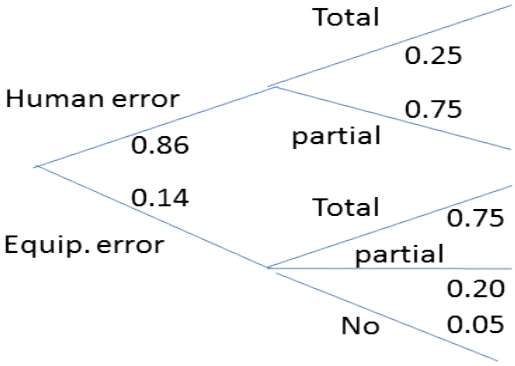
$$P(Y = 1) = C_1^3(0.1854)^1(1 - 0.1854)^2 = 0.36907$$

5) 86 % of process failures are due to human error, and the rest is due to equipment factors. Equipment factors result in a total shutdown 73% of the time, or a partial shutdown 20% of the time or no shutdown. On the other hand, a human error results in total shutdown 25% of the time, or no shutdown.

a) What is the probability of a total shutdown? *(3pts)*

$$P(total\ shutdown) = (0.14)(0.75) + (0.86)(0.25)$$

$$= 0.105 + 0.215 = 0.32$$



b) If the process is totally shutdown, what is the probability that the cause was human error? *(2pts)*

$$P(human\ error|total\ shutdown) = \frac{P(human\ error\ \cap\ total\ shutdown)}{P(total\ shutdown)}$$

$$P(human\ error|total\ shutdown) = \frac{0.215}{0.32} = \frac{43}{64} = 0.671875$$