King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 302 Major Exam 1
The Third Semester of 2012-2013 (123)

Time Allowed: 120 Minutes

Name: ______________________ ID#: ____________

Instructor: ____________________ Sec #: ______ Serial #: ______

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		14
2		12
3		16
4		12
5		15
6		16
7		15
Total		100

Q:1 (14 points) Find the solution of the initial value problem

$$\mathbf{r}''(t) = \sec^2 t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}, \text{ with } \mathbf{r}'(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} \text{ and } \mathbf{r}(0) = -\mathbf{j} + 5\mathbf{k}.$$

Q:2 (12 points) Let $f(x,y) = 2xy - x^2y$.

(a) Find the directional derivative of $f(x,y)$ at $(3,2)$ in the direction of a tangent vector to the graph of $3x^2 + 2y^2 = 11$ at $(1,2)$.

(b) Find the direction and value of maximum directional derivative of $f(x,y)$ at $(3,2)$.

Q:3 (16 points) Show that the vectors $\mathbf{u}_1 =< 1, 1, 1 >$, $\mathbf{u}_2 =< 2, 0, -1 >$ and $\mathbf{u}_1 =< 3, 1, 5 >$ are linearly independent. Also write the vector $\mathbf{v} =< 7, -1, 5 >$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$.

Q:4 (12 points) Let $A = \begin{bmatrix} 4 & -3 \\ x & -4 \end{bmatrix}$. Find value of x such that $A = A^{-1}$.

Q:5 (15 points) Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find an orthogonal matrix P such that $P^{-1}AP = D$, where D is an orthogonal matrix.

Q:6 (8+8 points) (a) Let $S = \{(x, y, x) \in \mathbb{R}^3 | 2x + 3y + 4z = 0\}$ be a subset of $3 - D$ space.

Show that S is a subspace of $\mathbb{R}^3$. Write the dimension of S.

(b) Let $S = \{(x, y, x) \in \mathbb{R}^3 | 2x + 3y + 4z = 12\}$ be a subset of $3 - D$ space. Determine whether S is a subspace of $\mathbb{R}^3$ or not.

Q:7 (15 points) Write **Correct** or **NotCorrect**. If not correct then write what is correct.

(A) Every symmetric matrix is diagonalizable but not orthogonally diagonalizable.

(B) Every matrix is diagonalizable.

(C) An $n \times n$ symmetric matrix *may* or *maynot* have n linearly independent eigenvectors.

(D) Rank of the coefficient matrix of a linear system is equal to the number of free variables in the solution of the system.

(E) For all matrices, the complex eigenvalues always occur in complex conjugate pairs.