Math 301-123 Quiz 3 A

Name:..Sec#:................ID#:..............................Ser#:.........

Q:1 (3+3+4 points) Find the following:

(a) $\mathcal{L}^{-1}\left\{\dfrac{3s-4}{s^2+8s+25}\right\}$,

(b) $\mathcal{L}\{f(t)\}$, where $f(t)=\begin{cases} \sin(t), & 0\le t<\pi \\ e^{2t}\cos(t), & t\ge\pi \end{cases}$.

(c) Solve the differential equation $y''+5y'-6y=\mathcal{U}(t-3)$ with $y(0)=0$ and $y'(0)=0$.

Sol: (a) $\mathcal{L}^{-1}\left\{\dfrac{3s-4}{s^2+8s+25}\right\}=\mathcal{L}^{-1}\left\{\dfrac{3(s+4)-16}{(s+4)^2+3^2}\right\}$

$$=\mathcal{L}^{-1}\left\{\frac{3(s+4)}{(s+4)^2+3^2}\right\}-\frac{16}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+4)^2+3^2}\right\}$$

$$=3e^{-4t}\cos 3t-\frac{16}{3}e^{-4t}\sin 3t$$

(b) $f(t)=\sin t-\sin t\,\mathcal{U}(t-\pi)+e^{2t}\cos t\,\mathcal{U}(t-\pi)$

$$\mathcal{L}\{f(t)\}=\frac{1}{s^2+1}-e^{-\pi s}\mathcal{L}\{\sin(t+\pi)\}+e^{-\pi s}\mathcal{L}\{e^{2t+2\pi}\cos(t+\pi)\}$$

$$=\frac{1}{s^2+1}+e^{-\pi s}\mathcal{L}\{\sin t)\}-e^{-\pi s}\mathcal{L}\{e^{2t+2\pi}\cos t\}$$

$$=\frac{1}{s^2+1}+e^{-\pi s}\frac{1}{s^2+1}-e^{-\pi s}\frac{e^{2\pi}(s-2)}{(s-2)^2+1}$$

(c) Taking Laplace of the given differential equation, we get

$$s^2Y(s)-sy(0)-y'(0)+5sY(s)-5y(0)-6Y(s)=\frac{e^{-3s}}{s}$$

$$(s^2+5s-6)Y(s)=\frac{e^{-3s}}{s}$$

$$\Rightarrow Y(s)=\frac{e^{-3s}}{s(s+6)(s-1)}$$

$$\Rightarrow Y(s)=e^{-3s}\left(\frac{-\frac{1}{6}}{s}+\frac{\frac{1}{42}}{s+6}+\frac{\frac{1}{7}}{s-1}\right)$$

$$\Rightarrow y(t)=-\tfrac{1}{6}\mathcal{U}(t-3)+\tfrac{1}{42}e^{-6(t-3)}\mathcal{U}(t-3)+\tfrac{1}{7}e^{t-3}\mathcal{U}(t-3)$$