King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Final Exam
The Third Semester of 2012-2013 (123)

(Version 1) **Time Allowed: 180 Minutes**

Name: ______________________________ ID#: ____________________

Instructor: ________________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.
- Write correct answer of MCQ on the front page.

MCQ Question #	Answer	Maximum Marks
1		7
2		7
3		7
4		7
5		7
6		7
7		7
8		7
Total		56

Written Question #	Marks	Maximum Marks
9		19
10		20
11		15
12		15
13		15
Total		84
Grand Total		140

Q:1 If $f(t) = e^{-2t}\cos^2 3t$ and $F(s) = \mathcal{L}\{f(t)\}$, then $F(S)$ is equal to:

(A) $\dfrac{1}{(s+2)} + \dfrac{s+2}{(s+2)^2+36}$

(B) $\dfrac{(s+2)^2}{((s+2)^2+36)^2}$

(C) $\dfrac{s^2+4s+22}{(s^2+4s+40)(s+2)}$

(D) $\dfrac{2s^2+8s+17}{2(s^2+4s+13)(s+2)}$

(E) $\dfrac{1}{2(s+2)} + \dfrac{s+2}{2(s+2)^2+9}$

Q:2 If $F(s) = \dfrac{e^{-3s}}{(s+1)^2+4} + e^{-4s}$, and $f(t) = \mathcal{L}^{-1}\{F(s)\}$, then $f(t)$ is equal to:

(A) $\frac{1}{2}\mathcal{U}(t-3)e^{t-3}\sin(2t-6) + \delta(t-4)$

(B) $\frac{1}{2}e^{3-t}\sin(2t-6) + \delta(t+4)$

(C) $\frac{1}{2}\mathcal{U}(t-3)e^{3-t}\sin(2t-6) + \dfrac{1}{t+4}$

(D) $\frac{1}{2}e^{t-3}\sin 2(t-6) + \dfrac{1}{t-4}$

(E) $\frac{1}{2}\mathcal{U}(t-3)e^{3-t}\sin(2t-6) + \delta(t-4)$

Q:3 The Fourier transform of $f(x) = e^{-2|x|}$ is:

(A) $\dfrac{4}{\alpha^2 + 4}$

(B) $\dfrac{\alpha}{\alpha^2 + 4}$

(C) $\dfrac{1}{\alpha^2 + 4}$

(D) $\dfrac{4}{\alpha^2 + 1}$

(E) $\dfrac{1}{\alpha^2 + 1}$

Q:4 The inverse Fourier transform of $F(\alpha) = e^{-3|\alpha|}$ is:

(A) $\dfrac{1}{\pi}\,\dfrac{3}{x^2 + 1}$

(B) $\dfrac{1}{\pi}\,\dfrac{3}{x^2 + 3}$

(C) $\dfrac{1}{\pi}\,\dfrac{9}{x^2 + 9}$

(D) $\dfrac{1}{\pi}\,\dfrac{3}{x^2 + 9}$

(E) $\dfrac{1}{\pi}\,\dfrac{3}{x^2 - 9}$

Q:5 If $\sum_{n=1}^{\infty} b_n \sin n\pi x$ is the Fourier sine series of $f(x) = x + 1$, $0 < x < 1$, then $b_1 + b_2 + 3b_3$ is equal to:

(A) $-\frac{11}{2\pi}$

(B) $\frac{11}{\pi}$

(C) $-\frac{1}{\pi}$

(D) $\frac{2}{\pi}$

(E) $\frac{12}{\pi}$

Q:6 If $f(x) = \sum_{n=0}^{\infty} c_n P_n(x)$ is the Fourier Legendre series of $f(x) = e^x$, $-1 < x < 1$, and $P_2(x) = \frac{1}{2}(3x^2 - 1)$, then c_2 is equal to:

(A) $\frac{5}{2}\left(e - 7e^{-1}\right)$

(B) $\frac{2}{3}\left(e - 5e^{-1}\right)$

(C) $\frac{3}{2}\left(7e - e^{-1}\right)$

(D) $\frac{5}{2}\left(e - e^{-1}\right)$

(E) $\frac{5}{2}\left(e + 7e^{-1}\right)$

Q:7 The Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq x < \pi \end{cases}$ is:

(A) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi} \sin nx$

(B) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi} \sin nx$

(C) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n + 1}{\pi} \cos nx$

(D) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi} \cos nx$

(E) $\frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi} \sin nx$

Q:8 If $y_n(x)$ and $y_m(x)$ are two eigenfunctions corresponding to two different eigenvalues of the Sturm–Liouville problem: $x^2y'' + xy' + \lambda y = 0$, $y(1) = 0$ and $y(5) = 0$, then which one is TRUE:

(A) $\int_1^5 x y_n(x) y_m(x) dx \neq 0$

(B) $\int_1^5 e^x y_n(x) y_m(x) dx = 0$

(C) $\int_1^5 \frac{1}{x} y_n(x) y_m(x) dx = 0$

(D) $\int_1^5 y_n(x) y_m(x) dx = 0$

(E) $\int_1^5 \frac{1}{x} y_n(x) y_m(x) dx \neq 0$

Q:9 (19 points) Find nontrivial solution of the heat equation $4\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}$ subject to the following initial and boundary conditions:

$u(0, t) = 0$, $u(\pi, t) = 0$, $t > 0$,

$u(x, 0) = 10$, $0 < x < \pi$.

Q:10 (20 points) Find nontrivial solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the following initial and boundary conditions:

$u(0, y) = 0$, $u(\pi, y) = 0$, $y > 0$,

$u(x, 0) = x$, $0 < x < \pi$ and solution is bounded as $y \to \infty$.

Q:11 (15 points) Let $u(r,z) = \sum_{n=1}^{\infty} A_n \sinh \alpha_n z \, J_0(\alpha_n r)$ is the steady state temperature in a cylinder of radius 2 obtained by solving the problem:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0 \;, \quad 0 < r < 2, \quad 0 < z < 4$$

$$u(2,z) = 0, \quad 0 < z < 4$$

$$u(r,0) = 0, \;\; u(r,4) = 4, \quad 0 < r < 2.$$

Apply the appropriate boundary condition and find A_n.

Q:12 (15 points) Solve the following problem using Laplace transform:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \quad t > 0,$$

subject to the following initial and boundary conditions:

$u(0,t) = \sin 3t, \quad \lim_{x\to\infty} u(x,t) = 0, \quad t > 0$

$u(x,0) = 0, \quad u_t(x,0) = 0, \quad x > 0.$

Q:13 (15 points) Solve the following problem using Fourier transform:

$k\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}, \quad -\infty < x < \infty, \quad t > 0$, subject to the condition:

$u(x, 0) = e^{-|x|}, \quad -\infty < x < \infty.$

Formula Sheet for Math 301-123 Final Exam

1. Two recurrence relations

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x), \qquad \frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$$

2. The Fourier Bessel series of f defined on the interval $(0, b)$ is $f(x) = \sum\limits_{i=1}^{\infty} c_i J_n(\alpha_i x)$, where $c_i = \dfrac{2}{b^2 J_{n+1}^2(\alpha_i b)} \int\limits_0^b x J_n(\alpha_i x) f(x) dx$ and α_i are defined by $J_n(\alpha b) = 0$.

3. The Fourier Bessel series of f defined on the interval $(0, b)$ is $f(x) = \sum\limits_{i=1}^{\infty} c_i J_n(\alpha_i x)$, where $c_i = \dfrac{2\alpha_i^2}{(\alpha_i^2 b^2 - n^2 + h^2) J_n^2(\alpha_i b)} \int\limits_0^b x J_n(\alpha_i x) f(x) dx$ and α_i are defined by $h J_n(\alpha b) + \alpha b J_n'(\alpha b) = 0$.

4. The Fourier Bessel series of f defined on the interval $(0, b)$ is $f(x) = c_1 + \sum\limits_{i=2}^{\infty} c_i J_0(\alpha_i x)$, where $c_1 = \dfrac{2}{b^2} \int\limits_0^b x f(x) dx$, $c_i = \dfrac{2}{b^2 J_0^2(\alpha_i b)} \int\limits_0^b x J_0(\alpha_i x) f(x) dx$ and α_i are defined by $J_0'(\alpha b) = 0$.

5. The Fourier-Legendre series of f defined on the interval $(-1, 1)$ is $f(x) = \sum\limits_{n=0}^{\infty} c_n P_n(x)$, where $c_n = \dfrac{2n+1}{2} \int\limits_{-1}^{1} f(x) P_n(x) dx$.