King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 2
The Third Semester of 2012-2013 (123)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		14
2		14
3		14
4		18
5		12
6		12
7		16
Total		100

Q:1 (7+7 points) Find the following Laplace transforms:

(a) $\mathcal{L}\{te^{-t}\sin^2 3t\}$,

(b) $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2+2s+3)}\right\}$.

Q:2 (14 points) Solve the integro-differential equation:

$$f(t) = 2t - \int_0^t (e^{2\tau} - e^{-2\tau}) f(t-\tau) d\tau$$

Q:3 (14 points) Solve the initial value problem using Laplace transform $y'' - y' - 2y = \delta(t - \pi)$ with $y(0) = 1$, $y'(0) = 1$.

Q:4 (3+3+12 points) Let $f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$.

(a) Sketch the graph of the function f.

(b) Sketch the graph of the Fourier series of f on separate coordinate axes.

(c) Find the Fourier series of f.

Q:5 (12 points) Let $f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & 2 \leq x < 3 \end{cases}$.

Find the Fourier sine series of f.

Q:6 (6+6 points) Consider the differential equation $y'' - 2y' = \lambda y,\ x \in (0,1)$ with the boundary conditions $y(0) = 0$ and $y(1) = 0$.

(a) Write the differential equation in self-adjoint form.

(b) Is this a regular Sturm–Liouville problem? If yes, write the weight function and associated inner product (orthogonality condition).

Q:7 (16 points) Find the first two terms of the eigenfunction expansion of $f(x) = \sin(\pi x)$ in eigenfunctions of the Sturm-Liouville problem

$y'' + \lambda y = 0,\ x \in (0,1)$ with $y'(0) = 0, y'(1) = 0.$