King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 1
The Third Semester of 2012-2013 (123)

Time Allowed: 120 Minutes

Name: ______________________________ ID#: ____________________

Instructor: ________________________ Sec #: __________ Serial #: __________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		12
3		10
4		12
5		12
6		14
7		15
8		15
Total		100

Q:1 (10 points) Find length of the curve traced by $\vec{r}(t) = e^{2t}\cos(3t)\,\mathbf{i} + e^{2t}\sin(3t)\mathbf{j} + e^{2t}\mathbf{k}$ on the interval $0 \le t \le 2\pi$. Also find a unit tangent vector to the curve at $t = \pi$.

Q:2 (12 points) Let $f(x, y) = 2xy - x^2y$.

(a) Find the directional derivative of $f(x, y)$ at $(3, 2)$ in the direction of a tangent vector to the graph of $x^2 + 2y^2 = 6$ at $(2, 1)$.

(b) Find the direction and value of maximum directional derivative of $f(x, y)$ at $(3, 2)$.

Q:3 (10 points) Let $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\vec{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ is a constant vector. Show that

(a) $\nabla \times [(\vec{r} \cdot \vec{r})\, \vec{a}] = 2(\vec{r} \times \vec{a})$

(b) $\nabla \cdot [(\vec{r} \cdot \vec{r})\, \vec{a}] = 2(\vec{r} \cdot \vec{a})$

Q:4 (12 points) Find work done by the force $\vec{F} = (2xy - e^{3y})\mathbf{i} + (x^2 - 3xe^{3y})\mathbf{j}$ along the curve $y = x^4$ for $0 \leq x \leq 1$.

Q:5 (12 points) Use Green's theorem to evaluate the line integral

$\oint_C (2x^2 - 2y^3)dx + (2x^3 + 3y^2)dy,$

$C = C_1 \bigcup C_2$, where C_1 is a positively oriented circle $x^2 + y^2 = 9$ and C_2 is a negatively oriented circle $x^2 + y^2 = 4$.

Q:6 (14 points) Evaluate the surface integral $\int\int_S (2xz + 3yz)\, dS$, where S is the portion of the plane $3x + 2y + 4z = 12$ in the first octant. Use projection of S onto yz-plane.

Q:7 (15 points) Use Stokes' theorem to evaluate the integral $\iint_S curl(F) \cdot \hat{n}\, dS$, where

$\vec{F} = \dfrac{xz}{4}\mathbf{i} + 4xy\mathbf{j} + 3xyz\mathbf{k}$ and S is the portion of the paraboloid $z = x^2 + 4y^2$

for $0 \leq z \leq 16$.

Q:8 (15 points) Use divergence theorem to evaluate $\iint_S (\vec{F}.\hat{n})\, dS$ where $\vec{F} = 6xz\mathbf{i} + 5y^2\mathbf{j} - 3z^2\mathbf{k}$ and D the region bounded by $z = y$, $z = 4 - y$, $z = 2 - \frac{1}{2}x^2$, $x = 0$ and $z = 0$.