KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 102 (123)
FINAL EXAM

Name:..ID:......................Section:....

Exercise 1: The integral

$$\int_{-2}^{2} (x^5 + 3)\sqrt{4 - x^2}dx$$

is equal to

(a) 6π

(b) 3π

(c) $\frac{\pi}{2}$

(d) $3 + 2\pi$

(e) $5 + 2\pi$

Exercise 2: If

$$F(x) = \int_{1}^{x^2} \frac{1}{2\sqrt{t}} \tan^{-1}\left(\sqrt{t}\right) dt \quad , \qquad x > 0$$

then $F(1) + F'(1) + F''(1)$ is equal to

(a) $\frac{\pi+2}{4}$

(b) $\frac{\pi-2}{4}$

(c) $\frac{2\pi-1}{2}$

(d) $\frac{\pi+1}{2}$

(e) 0

Exercise 3: The integral

$$\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}dx$$

is equal to

(a) $\frac{2}{3}\left[2-\frac{1}{x}\right]^{3/2}+C$

(b) $3\left[2-\frac{1}{x}\right]^{1/3}+C$

(c) $2\left[2-\frac{1}{x}\right]^{1/2}+C$

(d) $\frac{1}{3}\left[2-\frac{1}{x}\right]^{3/2}+C$

(e) $\left[2-\frac{1}{x}\right]^{3/2}+C$

Exercise 4: The area of the region below the line $y=1$ and between the curves $y=\tan x$ and $x=0$ is equal to

(a) $\frac{1}{4}(\pi-2\ln 2)$
(b) $\frac{1}{4}(\pi+\ln 2)$
(c) $\frac{\pi}{4}-1$
(d) $\frac{\pi}{2}+\ln 2$
(e) $\frac{\pi}{4}-\ln 2$

Exercise 5: The volume of the solid generated by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the line $x = -1$ is given by

(a) $\int_0^1 \pi \left[(y+1)^2 - (y^2+1)^2\right] dy$
(b) $\int_0^1 \pi \left[(y-1)^2 - (y^2+2)^2\right] dy$
(c) $\int_0^1 \pi \left[y^2 - y^4\right] dy$
(d) $\int_0^1 \pi \left[(y-1)^2 - (y^2-1)^2\right] dy$

(e) $\int_0^1 \pi \left[y^4 - y^2 + 2y - 1\right] dy$

Exercise 6: The region bounded by the curve $y = 2\sqrt{x}$, the $x-$axis, and the line $x = 4$ is rotated about the $y-$axis. The volume of the solid generated is equal to

(a) $\frac{2^8}{5}\pi$
(b) $\frac{2^6}{3}\pi$

(c) $\frac{2^8}{3}\pi$

(d) $\frac{2^6}{5}\pi$

(e) $\frac{2^6}{4}\pi$

Exercise 7: The sum of the series

$$\frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \cdots$$

is equal to
(a) $2 - \ln 3$
(b) 3
(c) $\ln 3$
(d)$1 + \ln 3$
(e) 2

Exercise 8: The length of the curve

$$y = \int_0^x \sqrt{\sec^4 t - 1} dt \ , \ 0 \leq x \leq \frac{\pi}{4}$$

is equal to
(a) 1
(b) $\sqrt{2}$
(c) $3 - \sqrt{2}$
(d) 2
(e) $1 + \sqrt{3}$

Exercise 9: The area of the surface obtained by rotating the curve $y = \cosh x$, $0 \le x \le 1$ about the y-axis is given by

(a) $\int_0^1 2\pi x \cosh x dx$
(b)$\int_0^1 2\pi x \text{sinh} x dx$
(c) $\int_0^1 2\pi \cosh x \text{sinh} x dx$
(d) $\int_0^1 2\pi x \cosh x \text{sinh} x dx$
(e) $\int_0^1 2\pi x \text{sech} x dx$

Exercise 10: The integral

$$\int \frac{dx}{2\sqrt{x} + 2x}$$

is equal to

(a) $\ln(1 + \sqrt{x}) + C$
(b) $\ln(\sqrt{x}) + C$

(c) $2\ln(1 + \sqrt{x}) + C$

(d) $\frac{1}{2}\ln(1 + \sqrt{x}) + C$

(e) $\ln(x + \sqrt{x}) + C$

Exercise 11: The integral

$$\int_0^1 e^t \cosh t dt$$

is equal to
(a) $\frac{1}{4}(e^2+1)$
(b) $\frac{1}{2}(e^2+1)$
(c) e^2-1
(d) $\frac{1}{2}(e-2)$
(e) $\frac{1}{3}(e+2)$

Exercise 12: The integral

$$\int_0^{\frac{\pi}{2}} \mathrm{e}^{\cos x} \sin(2x) dx$$

is equal to
(a) 2
(b) -$\frac{1}{2}$
(c) 0
(d) 3
(e) -4

Exercise 13: The integral

$$\int \frac{\sin^3 x}{\cos^4 x} dx$$

is equal to

(a) $\frac{1}{3}\sec^3 x - \sec x + C$
(b) $3\sec x - \frac{1}{3}\sec^3 x + C$

(c) $\sec^3 x + \sec x + C$

(d) $\sec^2 x + \frac{1}{3}\sec^3 x + C$

(e) -$\sec^3 x + \frac{1}{3}\sec x + C$

Exercise 14: If $x > 2$, the integral

$$\int \frac{\sqrt{x^2-4}}{x} dx$$

is equal to
(a) $\sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$
(b) $\frac{2\sqrt{x^2-4}}{x} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$
(c) $2\sqrt{x^2-4} + 2\tan^{-1}\left(\frac{x}{2}\right) + C$
(d) $x - 2\sec^{-1}\left(\frac{x}{2}\right) + C$
(e) $2x\sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$

Exercise 15: If

$$\frac{3x^2+2x+1}{(x-1)(x^2+2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5}$$

then $A+B+C$ is equal to
(a) $\frac{23}{4}$
(b) $\frac{12}{5}$
(c) $\frac{21}{4}$
(d) $\frac{17}{5}$
(e) 0

Exercise 16: The integral

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x} dx$$

is equal to
(a) $\frac{1}{2}\ln|x| + \frac{1}{10}\ln|2x-1| - \frac{1}{10}\ln|x+2| + C$
(b) $\ln|x| + \ln|2x-1| - \ln|x+2| + C$
(c) $\frac{1}{2}\ln|x| - \ln|x+2| + C$
(d) $\frac{1}{2}\ln|x| + \frac{1}{5}\ln|2x-1| + 3\ln|x+2| + C$
(e) $2\ln|x| + 3\ln|2x-1| - 3\ln|x+2| + C$

Exercise 17: The integral

$$\int_2^\infty \frac{dx}{x(\ln x)^p}$$

converges for
(a) $p > 1$
(b) $p = 0$
(c) $p = -1$
(d) $p = 1$
(e) $p < 1$

Exercise 18: The sequence

$$\left\{ \left(1 - \frac{2}{5n}\right)^{5n} \right\}_{n \geq 1}$$

is
(a) convergent and its limit is e^{-2}
(b) convergent and its limit is e^{-5}
(c) convergent and its limit is $e^{-2/5}$
(d) convergent and its limit is e^{3}
(e) divergent

Exercise 19: The series

$$\sum_{n\geq 1} \frac{\sin(n\pi) + 2^n}{3^n}$$

is equal to
(a) 2
(b) 3
(c) 33
(d) $\frac{2}{33}$
(e) 6

Exercise 20: Which of the following proposition is True about the series

$$\sum_{n\geq 1} \frac{n}{n^2+1}$$

(a) Diverges by the integral test
(b) Converges by the integral test

(c) Converges by the nth term test
(d) Diverges by the nth term test

(e) The integral test cannot be applied

Exercise 21: The series

$$\sum_{n\geq 1} \frac{4}{n(n+2)}$$

is equal to
(a) 3
(b) 4
(c) 5
(d) 7
(e) 1024

Exercise 22: The series

$$\sum_{n\geq 1} \left(\frac{3n}{4n+1}\right)^n$$

is

(a) Convergent by the root test
(b) Divergent by the root test

(c) A series for which the root test is inconclusive
(d) Divergent by the nth-term test of Divergence
(e) Divergent by the limit comparison test

Exercise 23: The series

$$\sum_{n\geq 1}^{n} \frac{3^{n+2}}{\ln n}$$

is

(a) Divergent by the ratio test
(b) Convergent by the ratio test

(c) A series for which the ratio test is inconclusive
(d) Convergent by the nth root test
(e) The ratio test cannot be applied

Exercise 24: Which of the following proposition is False about the series

$$\sum_{n\geq 1} (-1)^{n+1} \frac{n}{n^3+1}$$

(a) Diverges
(b) Converges absolutely

(c) Converges
(d) Converges by the alternating series test
(e) converges with the absolute convergence test and the alternating series test

Exercise 25: Which of the following proposition is True about the series

$$\sum_{n\geq 1}(-1)^{n+1}\frac{n!}{2^n}$$

(a) Diverges
(b) Converges absolutely

(c) Converges by the ratio test
(d) Converges by the integral test
(e) converges absolutely by the ratio test

Exercise 26: The interval of convergence of the power series

$$\sum_{n\geq 1}\left(1+\frac{1}{n}\right)^n (x-1)^n$$

is

(a) $(0, 2)$
(b) $[0, 2]$

(c) $\left(\frac{e-1}{e}, \frac{e+1}{e}\right)$
(d) $[-1, 3)$

(e) $[0, 2)$

Exercise 27: The interval of convergence of the power series

$$\sum_{n\geq 1}^{n} \frac{5^n}{n}(x+1)^n$$

is

(a) $[-\frac{6}{5}, -\frac{4}{5})$
(b) $(-\frac{6}{5}, -\frac{4}{5})$

(c) $[-\frac{6}{5}, -\frac{4}{5}]$
(d) $(-2, 1)$

(e) $[-2, 1)$

Exercise 28: The Taylor series of $e^{2x} \sin x$ is

(a) $x + 2x^2 + \frac{11}{6}x^3 + ...$
(b) $x - 3x^2 + \frac{1}{6}x^3 + ...$

(c) $x + 2x^2 + \frac{11}{5}x^3 + ...$
(d) $x - 3x^2 + \frac{11}{6}x^3 + ...$

(e) $x + 2x^2 - \frac{11}{5}x^3 + ...$