King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Stat 319: Probability and Statistics for Engineers and Scientists
Major Exam 2 – Spring 2013 (T122)
Monday, April 15, 2013
Allowed Time: 75 minutes, from 6:15-7:30pm

Instructors: (Circle One)

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Name: KEY ID #: 0000

Section #: ____________ Serial Number: ______________

Instructions:

1. **Show all your work and write clearly.** No points for answers without justification!
2. **Only basic calculators are allowed.**
3. **Cell-phones should be turned off.**
4. Do not copy from or communicate with any other person.

Question	Score	Points
1		11
2		11
3		16
4		6
5		6
6		10
Total:		60

1. A high-volume printer produces minor print-quality errors on a test pattern of 1000 pages of text according to a Poisson distribution with a mean of 0.4 per page.

   (a) [5 points] What is the mean number of pages with errors (one or more)?

$\lambda = 0.4$, X: # of errors per page. $X: Po(0.4)$

① $f(x) = \begin{cases} \frac{e^{-0.4}(0.4)^x}{x!}, & x = 0,1,2,\cdots \\ 0, & o.w. \end{cases}$

① $P(X \geq 1) = 1 - P(X=0) = 1 - \frac{e^{-0.4}(0.4)^0}{0!}$

$= 1 - 0.6703 = \boxed{0.3297}$ ①

Let Y: # of pages with one or more errors.

① $Y: B(n,p)$ where $n = 1000$, $p = 0.3297 \approx 0.33$

$\Rightarrow \mu_Y = E(Y) = np = 1000(0.33) = \boxed{330}$ pages. ①

   (b) [6 points] Approximate the probability that more than 350 pages contain errors (one or more).

$Y: B(1000, 0.33) \approx N(\mu_Y, \sigma_Y^2)$ where

① $\mu_Y = np = 330$, $\sigma_Y = \sqrt{npq} = 14.869 \approx 14.87$ ①

$\Rightarrow P(Y > 350) \approx P(Y > 350.5)$ ① ①

$\approx P\left(\frac{Y - \mu_Y}{\sigma_Y} > \frac{350.5 - 330}{14.869}\right)$

$\approx P(Z > 1.38) =$ ①

$\approx \Phi(-1.38)$

$\approx \boxed{0.0838}$ ①

2. The waiting time (in minutes) for an elevator at the basement of a bank has the p.d.f.

$$f(x) = \begin{cases} e^{-x}, & x > 0; \\ 0, & \text{otherwise.} \end{cases}$$

(a) 4 points] Find the median waiting time.

Median $= m$ such that $P(X \leq m) = P(X \geq m) = \frac{1}{2}$ 1

where $X : Exp(1) \Rightarrow F(x) = 1 - e^{-x}$ 1

$\Rightarrow F(m) = 1 - e^{-m} = \frac{1}{2} \Rightarrow e^{-m} = \frac{1}{2}$ 1 $\Rightarrow$

$-m = -\ln 2 \Rightarrow m = \boxed{\ln 2 = 0.6931}$

1

(b) [3 points] Find the probability that the elevator doesn't come within 3 minutes.

$P(X > 3) = e^{-3} = 0.0497 \approx 0.05$

1 1 1

(c) 4 points] Find the probability that you have to wait at least three more minutes if you have already waited two minutes.

1 $P(X > 5 | X > 2) = \frac{P(X > 2 \cap X > 5)}{P(X > 2)}$ 1

$= \frac{P(X > 5)}{P(X > 2)} = \frac{e^{-5}}{e^{-2}} = \frac{0.0067}{0.1354} = \boxed{0.0498}$ 1 1

or simply, $P(X > 5 - 2) = P(X > 3) = \boxed{0.0498}$

(2) (1) (1)

3. The tragedy that befell the space shuttle Challenger and its astronauts in 1986 led to a number of studies to investigate the reasons for mission failure. Attention quickly focused on the behavior of the rocket engine's O-rings. Here is data consisting of observations on O-ring Temperature (F°) for each test firing or actual launch of the shuttle rocket engine.

31	40	45	49	52	53	57	58	58	60	61	61	63	66	67	67	67	68
68	69	70	70	70	70	72	73	75	75	76	76	78	79	80	81	83	84

$\sum x = 2372$

$\sum x^2 = 161466$

(a) 7 points Calculate the mean, mode, median, third quartile , interquartile range.

$\bar{x} = \frac{\sum x}{n} = \frac{2372}{36} = \boxed{65.8889}$ 1

1 mode = $\boxed{70}$, 1 median = $\tilde{x} = \boxed{68} = \frac{68+68}{2}$ 1

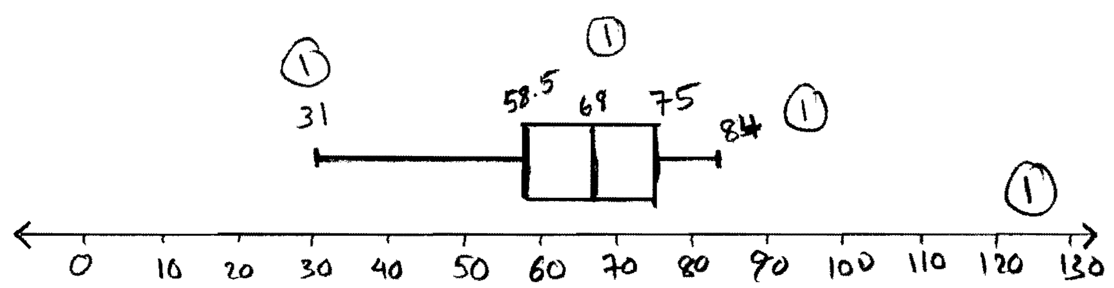
Given [Location $Q_1 = \frac{1}{4}(n+1) = \frac{1}{4}(37) = 9.25 \Rightarrow$

$Q_1 = x_{(9)} + \frac{1}{4}(x_{(10)} - x_{(9)}) = 58 + \frac{1}{4}(2) = \boxed{58.5}$]

1 Location $Q_3 = \frac{3}{4}(37) = 27.75 \Rightarrow Q_3 = \boxed{75}$ 1

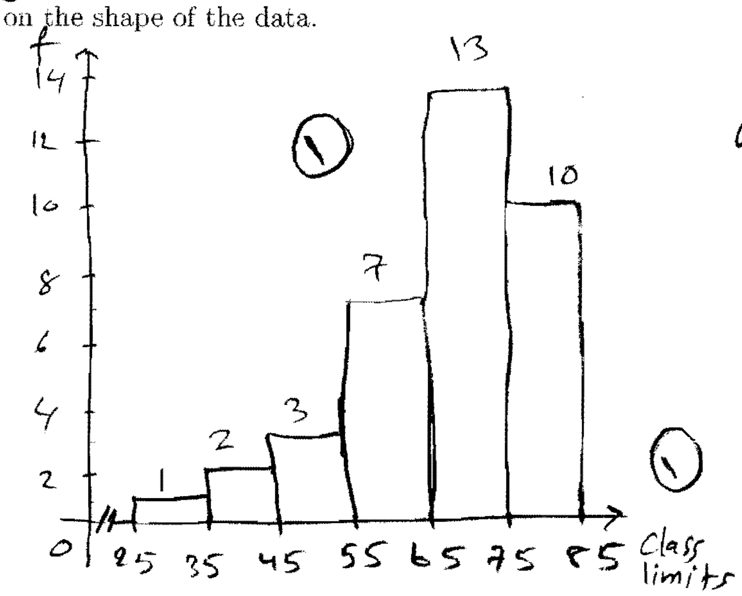
$IQR = Q_3 - Q_1 = 75 - 58.5 = \boxed{16.5}$ 1

(b) 4 points Construct a box plot of the data.



(c) 5 points Construct a histogram, starting from 25 and with a step size 10. Comment on the shape of the data.

1 Class	f
[25-35)	1
[35-45)	2
[45-55)	3
[55-65)	7
[65-75)	13
[75-85)	10
Total	36



The data are:

a. Unimodal & 1

b. Left skewed 1

4. **6** [points] Suppose that the breaking strength of cables (measured in pounds) is known to have a normal distribution with a standard deviation 6 pounds. How large a sample must be taken so as to be 90 percent confident that the sample mean breaking strength will not differ from the true mean breaking strength by more than 2 pounds?

X: Breaking strength (in pounds), $X : N(\mu, 36)$

$n \geq \frac{z_{\alpha/2}^2 \sigma^2}{E^2}$, where $E = 2$ ①

$1-\alpha = 0.9 \Rightarrow \frac{\alpha}{2} = \frac{0.1}{2} = 0.05$ ① $\Rightarrow z_{0.05}$ ① $= 1.64$

$\Rightarrow n \geq \frac{(1.64)^2 36}{4} = 24.2064 \approx 25$ ① ① ①

5. **6** [points] The charge-to-tap time (min) for a carbon steel in one type of open hearth furnace was determined for each heat in a sample of size 36, resulting in a sample mean time of 382.1 and a standard deviation of 31.5. Calculate a 96% confidence interval for the true average charge-to-tap time. Interpret this interval.

X: Charge-to-tap time (min), $n = 36$, $\bar{x} = 382.1$ min.

① $S = 31.5$ min, $1-\alpha = 0.96 \Rightarrow \frac{\alpha}{2} = \frac{0.04}{2} = 0.02$ ①

$\Rightarrow z_{\frac{\alpha}{2}} = z_{0.02} = 2.05$ ① $\Rightarrow$

A $(1-\alpha)100\%$ C.I. for μ is $\bar{x} \pm z_{\alpha/2} \frac{S}{\sqrt{n}}$

using the CLT. $\Rightarrow$

①

A 96% C.I. for μ is $382.1 \pm (2.05)\frac{31.5}{\sqrt{36}}$

$= [382.1 \pm (2.05)(5.25)]$ ①

$= [382.1 \pm 10.7625] = [371.3375, 392.8625]$

① We are 96% Confident that the true average Charge-to-tap time is between 371.3375 min & 392.8625 min.

6. The Arizona Department of Transportation wishes to survey state residents to determine what proportion of the population would like to increase statewide highway speed limits to 120 kmph from 100 kmph. To accomplish this task, it plans on using a telephone poll of randomly chosen residents.

(a) [4 points] How many residents do they need to survey if they want to be 99 percent certain that its estimate is correct to within ±0.05?

Since we have no $\hat{p}$ we estimate $\hat{p} = \frac{1}{2}$ 1

$\Rightarrow n \leq \frac{Z_{\frac{\alpha}{2}}^2}{E^2} \frac{1}{4} \Rightarrow Z_{\frac{0.01}{2}} = Z_{0.005} = 2.575$ 1

$\Rightarrow n \leq \left(\frac{2.575}{0.05}\right)^2 \frac{1}{4} = \frac{10609}{16} = 663.0625$ 1

$\boxed{663}$ This is the MAXIMUM sample size we need to draw. 1

(b) [6 points] Suppose there is a sample whose size is the answer in part (a). If 23 percent of the sample were like to increase statewide highway speed limits to 120 kmph from 100 kmph, construct and interpret a 98 percent confidence interval for the proportion of the residents that would like to increase statewide highway speed limits to 120 kmph from 100 kmph.

1 $\hat{p} = 0.23 \Rightarrow \hat{q} = 0.77,\ n = 663 \Rightarrow$ 1

check assumptions: $n\hat{p} = (10609)(0.23) = 2440.07 \geq 5$ ✓
$n\hat{q} = (10609)(0.77) = 8168.93 \geq 5$ ✓

$\Rightarrow$ A $(1-\alpha)100\%$ C.I. for p is $\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

where $Z_{\frac{0.02}{2}} = Z_{0.01} = 2.33$ 1 and 1

A 98% C.I. for p is $\left[0.23 \pm (2.33)\sqrt{\frac{(0.23)(0.77)}{663}}\right]$

$= [0.23 \pm (2.33)(0.0163)] = [0.23 \pm 0.0381]$ 1

Good Luck! ☺

$= [0.1919, 0.2681]$, that is 1

we are 98% Confident that the true proportion of the residents that would like to increase the speed limits is between 0.1919 & 0.2681.