T122 Quiz 5 on Hwk STAT301: A First Course in Probability FORM A

Name____ ID#:__________ Serial #:___

Instructions. The quiz is 20 minutes. Write **important steps** to arrive at the solution of the following **3** problems.

1. (7 marks) If X and Y are independent and identically distributed with mean $\mu = 4$ and variance $\sigma^2 = 25$, find $E[2(X-Y)^2]$

2. (7 marks) Let $(X_i, Y_i), i = 1, ...,$ be a sequence of independent and identically distributed random vectors. That is, X_1, Y_1 is independent of, and has the same distribution as X_2, Y_2 and so on. Although X_i and Y_i can be dependent, X_i and Y_j are independent when $i \neq j$. Let

$$\mu_x = E[X_i] = 3, \qquad \mu_y = E[Y_i] = 4, \qquad \sigma_x^2 = Var\,(X_i) = 16,$$
$$\sigma_y^2 = Var\,(Y_i) = 25, \qquad \rho = Corr(X_i, Y_i) = 0.8$$

Find $Corr\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{n} Y_j\right)$.

3. (2+4=6 marks) *The Conditional Covariance Formula.* The conditional covariance of X and Y, given Z, is defined by $\mathrm{Cov}(X, Y|Z) \equiv E[(X - E[X|Z])(Y - E[Y|Z])|Z]$ and $Cov(X, Y) = E[Cov(X, Y|Z)] + Cov(E[X|Z], E[Y|Z])$

(a) Show that $\mathrm{Var}(X|Z) = E[X^2|Z] - (E[X|Z])^2$

(b) Suppose passenger arrival at a train depot is a Poisson random variable T with mean $\lambda t = 10t$. The train arrival time at the depot is independent of passenger arrival and is a uniform random variable distributed over (0,T). What is the variance of the number of passengers who enter the train?.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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$$E[2(X-Y)^2]$$

2. (6 marks) *The Conditional Covariance Formula.* The conditional covariance of X and Y, given Z, is defined by $\text{Cov}(X, Y|Z) \equiv E[(X - E[X|Z])(Y - E[Y|Z])|Z]$

(a) Show that $\text{Cov}(X, Y|Z) = E[XY|Z] - E[X|Z]E[Y|Z]$

(b) Prove the conditional covariance formula $\text{Cov}\ (X, Y) = E[Cov(X, Y|Z)] + Cov(E[X|Z], E[Y|Z])$

(c) Set X and Y in part (b) and obtain the conditional variance formula

3. (7 marks) Let $(X_i, Y_i), i = 1, ...,$ be a sequence of independent and identically distributed random vectors. That is, X_1, Y_1 is independent of, and has the same distribution as X_2, Y_2 and so on. Although X_i and Y_i can be dependent, X_i and Y_j are independent when $i \neq j$. Let

$$\mu_x = E[X_i], \qquad \mu_y = E[Y_i], \qquad \sigma_x^2 = Var\left(X_i\right),$$
$$\sigma_y^2 = Var\left(Y_i\right), \qquad \rho = Corr(X_i, Y_i)$$

Find $\text{Corr}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{n} Y_j\right)$.