Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

STAT301: Introduction to Probability Theory

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Final Exam Term 122 FORM A
Wednesday May 22 2013 8.00am-10.30am

Name____ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile phone calculators, I-pad, or communicable devices are *disallowed*. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	3+2=5		
2	5		
3	5		
4	4+3+3=10		
5	5		
6	5		
7	5		
8	5		
9	5		
Total	50		
10 bonus	5		
11 bonus	1+4=5		
Total	60		

Extra blank page

1 (3+2=5 points) An insurance company offers home owners fire or theft insurance. 25% of its home owners have fire insurance, 60% have theft insurance and 11% have both.

a) What percentage of home owners have insurance that the company can offer?

b) Of the homes that have theft insurance, what percentage has fire insurance?

2. (5 points) Suppose that by any time t the number of people that have arrived at a bus depot is a Poisson random variable with mean λt. If the initial bus arrives at the depot at a time (independent of when the passengers arrive) that is uniformly distributed over $(0, T)$, what are the mean and variance of the number of passengers who enter the bus?

3. (5 points) If X is a standard normal random variable, find the moment generating function of $Y = X^2$, and use it to find the mean of Y.

4. (4+3+3=10 points) If X and Y have joint density function $f_{X,Y}(x, y) = \begin{cases} 1/y & \text{if } 0 < y < 1, \quad 0 < x < y \\ 0 & \text{otherwise} \end{cases}$.

(a) Find $E[XY]$.

(b) Find $E[X]$.

(c) Find $E[Y]$.

5. (5 points) Let $X_1, X_2, \cdots X_n$ be independent exponential random variables with $\lambda = 2$, representing insurance claim amounts (in thousands) for individuals 1 through n. Find $M_S(t)$, the **moment generating function** of the sum of these claim variables, $S = \sum_{i=1}^{n} X_i$. (Hint: $M_{X_i}(t) = \frac{\lambda}{\lambda - t}$).

6. (5 points) For a standard normal random variable Z, let $\mu_n = E[Z^n]$.

Show that $\mu_n = \begin{cases} 0 & \text{when } n \text{ is odd} \\ \frac{(2j)!}{(2^j j)!} & \text{when } n = 2j \text{ (i.e. } n \text{ is even).} \end{cases}$

(Hint: You may use Taylor series expansion about zero of $e^{t^2/2} = \sum_{j=0}^{\infty} \frac{t^{2j}}{2^j j!}$)

7. (1+4=5 points) A tobacco company claims that the amount of nicotine in one of its cigarettes is a random variable with mean 2.2 mg and standard deviation 0.3 mg. However, the average nicotine content of 100 randomly chosen cigarettes was 3.1 mg. Approximate the probability that the average nicotine content would have been **as high as or higher than** 3.1 if the company's claims were true.

8. (5 points) Let X be a Poisson random variable with parameter λ. Use Chebyshev's inequality to show that $P(X \leq \lambda - 2) < \frac{\lambda}{4}$.

9. (5 points) If X is a gamma random variable with parameters $(n, 1)$, approximately how large need n be so that $P\left(\left|\frac{X}{n} - 1\right| > 0.01\right) < 0.05$?

10. (Bonus 5 points) A sample of 3 items is selected at random from a box containing 9 items of which 5 are defective. What is the probability that **at most one** item in the sample is defective?.

11. (Bonus: 1+4 = 5 points) A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x,y) = \begin{cases} 6[1-(x+y)] & \text{for } x>0, \quad y>0, \quad x+y<1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the probability that the portion of a claim representing damage to the house is less than 0.2.
A) 0.360
B) 0.480
C) 0.488
D) 0.512
E) 0.520
Work shown (4 points)

Hence the answer is (___)

END OF TEST PAPER