Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

STAT301: Introduction to Probability Theory
Dr. Mohammad H. Omar
Major 2 Exam Term 122 FORM A
Sunday April 21 2013
6.00pm-7.30pm

Name_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile phone calculators, I-pad, or communicable devices are *disallowed*. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	5		
2	5		
3	5+5=10		
4	5		
5	4+3+3=10		
6	3+4+3=10		
7	3+3+4=10		
8	5		
Total	60		

Extra blank page

1 (5 points) A purchaser of electrical components buys them in lots of size15. It is his policy to inspect 4 components randomly from a lot and to accept the lot only if all 4 are nondefective. If 30 percent of the lots have 5 defective components and 70 percent have only 1, what percentage of lots does the purchaser reject?

2. (5 points) Verify that $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$ for $-\infty < x < \infty$, $\sigma > 0$, is a density function.

(Hint: define $I = \int_{-\infty}^{\infty} \exp(-w^2/2)dw$ and proceed).

3. (5+5 = 10 points) An accident loss random variable, T, for a policy-holder follows an exponential random variable with mean $\frac{1}{\lambda} = 2$ years.

   a) Find the **probability** that the policy holder will encounter an accident **within** the next 3 years.

   b) Find $Var(4 + 0.5T)$.

4. (5 points) Let $Y = \ln(X)$ be a normally distributed random variable with parameters $\mu = \ln 20.905$ and σ^2. In addition, $P(\ln(X) > 14.56) = 0.10$. Find the density function of X.

5. (4+3+3=10 points) Suppose the hazard rate function of the random variable X is given by $\lambda(t) = 2t$, $t > 0$.

(a) Find the cumulative distribution function, $F_X(x)$

(b) Calculate $P(1 < X < 3)$.

(c) Compute $E(X)$.

6. (3+4+3=10 points) Given that the density of X and Y is

$$f_{X,Y}\ (x,y) = 3e^{-(3x+y)} \text{ for } x > 0,\ y > 0.$$

a) Find $F_{X,Y}\ (x,y)$

b) Find $f_X(x)$

c) Find $f_{Y|X}(y|x)$

7. (1+4=5 points) A device containing two key components fails when, and only when, both components fail. The lifetimes, T_1 and T_2, of these components are independent with common density function $f(t) = e^{-t}$, $t > 0$. The cost, X, of operating the device until failure is $2T_1 + T_2$. Which of the following is the **density function** of X for $x > 0$.

A) $e^{-x/2} - e^{-x}$
B) $2\left(e^{-x/2} - e^{-x}\right)$
C) $\dfrac{x^2 e^{-x}}{2}$
D) $\dfrac{e^{-x/2}}{2}$
E) $\dfrac{e^{-x/3}}{3}$
Work shown (4 points)

END OF TEST PAPER