The one-sample problem

<u>p-value approach</u>

Hypothesis type	p-value	
Lower tail	$P(Z < z)$	$P(T < t)$
Upper tail	$P(Z > z)$	$P(T > t)$
2-tailed	$2P(Z > \|z\|)$	$2P(T > \|t\|)$

<u>Test statistics</u>

$z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$ or $t_{n-1} = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ will be

For testing hypotheses about *π*:

$z = \dfrac{p - \pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}}$ where $p = \dfrac{x}{n}$

For testing hypotheses about *σ*

Test statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

The two-sample problem

The i[th] **paired difference** $d_i = x_{1i} - x_{2i}$.

$$t_{n-1} = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$

$\bar{d} = \dfrac{\sum_{i=1}^{n} d_i}{n}$ and $s_d = \sqrt{\dfrac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}}$.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$$t_{n_1+n_2-2} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

where $s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$$t_{df} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where $df = \dfrac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(\dfrac{\left(s_1^2/n_1\right)^2}{n_1-1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2-1}\right)}$

To test hypotheses about *π*1 - *π*2

If $\bar{p} = \dfrac{n_1p_1 + n_2p_2}{n_1+n_2} = \dfrac{x_1+x_2}{n_1+n_2}$ then the test statistic is $z = \dfrac{(p_1-p_2)-(\pi_1-\pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$

For testing hypotheses about σ_1^2 and σ_2^2

Test statistic $F_0 = \dfrac{s_1^2}{s_2^2}$ with $df_1 = n_1 - 1$ $df_2 = n_2 - 1$.

Test for *c* Independent Proportions

If $\chi^2 = \sum_{j=1}^{c}\dfrac{(o_j - e_j)^2}{e_j} > \chi_\alpha^2$ [with df =c-1]

Marascuillo's Test for Pair-wise Proportions

If $|p_i - p_j| > \sqrt{\chi_\alpha^2}\sqrt{\dfrac{p_i(1-p_i)}{n_i} + \dfrac{p_j(1-p_j)}{n_j}}$

Test of Independence in *rxc* table

Test statistic $\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\dfrac{(o_{ij} - e_{ij})^2}{e_{ij}}$

Sample correlation coefficient

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}} = \frac{Sxy}{\sqrt{SxxSyy}}$$

For testing H_0: ρ=0 vs. H_A: ρ≠0

<u>Test statistic</u> $t_{n-2} = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}}$

Estimated regression model $\hat{y}_i = b_0 + b_1 x$

The Least Square Estimates are

$$b_1 = \frac{Sxy}{Sxx}$$

And $b_0 = \bar{y} - b_1 \bar{x}$

Total Sum of Squares

$$SST = Syy = \sum (y - \bar{y})^2 = \sum_1^n y_i^2 - n\bar{y}^2$$

$$SSR = \frac{(Sxy)^2}{Sxx} \quad SSE = SST - SSR$$

Coefficient of Determination

$$R^2 = \frac{SSR}{SST}$$

Standard Error of the model

$$s_\varepsilon = \sqrt{\frac{SSE}{n - k - 1}}$$

Standard Error of the Slope

$$s_{b_1} = \frac{s_\varepsilon}{\sqrt{Sxx}}$$

For testing H_0: $\beta_1 = 0$ vs. H_1: $\beta_1 \neq 0$

The test statistic & **C.I. for the slope**

$$t_{n-2} = \frac{b_1 - \beta_1}{s_{b_1}} \quad \& \quad b_1 \pm t_{\alpha/2} s_{b_1}$$

C.I. for the **mean** of y given a particular x_p

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x - \bar{x})^2}}$$

P.I. estimate for an **Individual value of y** given a particular x_p

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x - \bar{x})^2}}$$

For testing

H_0: $\beta_1 = \beta_2 = \ldots = \beta_k = 0$ H_A: at least one $\beta_i \neq 0$

Test statistic $F = \frac{MSR}{MSE}$

For testing H_0: $\beta_{j+1} = \beta_{j+2} = \ldots \ \beta_{j+m} = 0$

against H_A: at least one $\beta_i \neq 0$

Test statistic $F = \dfrac{\dfrac{SSR_{Full} - SSR_{Reduced}}{m}}{MSE_{Full}}$

Simple Index number formula & Unweighted aggregate price index formula (respectively)

$$I_t = \frac{y_t}{y_0} 100 \quad \& \quad I_t = \frac{\sum p_t}{\sum p_0} 100$$

Weighted Aggregate Price Indexes

Paasche $I_t = \dfrac{\sum q_t p_t}{\sum q_t p_0} 100$

Laspeyres $I_t = \dfrac{\sum q_0 p_t}{\sum q_0 p_0} 100$

Single Exponential Smoothing Model

$F_{t+1} = F_t + \alpha(y_t - F_t) = \alpha y_t + (1 - \alpha) F_t$ where

α: smoothing constant.

Exponential Trend Model $Y_t = \beta_o \beta_1^{X_t} \varepsilon_t$

Transformed Exponential Trend Model

$\log(Y_t) = \log(\beta_o) + X_t \log(\beta_1) + \log(\varepsilon_t)$

Exponential Model for Quarterly data

$Y_t = \beta_o \beta_1^{X_t} \beta_2^{Q_1} \beta_3^{Q_2} \beta_4^{Q_3} \varepsilon_t$

***p*th-order Autoregressive Model**

$Y_t = A_o + A_1 Y_{t-1} + A_2 Y_{t-2} \cdots + A_p Y_{t-p} + \varepsilon_t$

where A_p: the pth autoregressive parameter